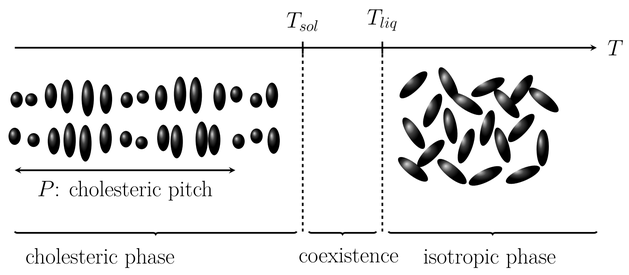
Texture of cholesteric droplets: finite element simulations and experiments

Guilhem Poy, Félix Bunel, Patrick Oswald

Univ Lyon, Ens de Lyon, Université Claude Bernard Lyon 1, CNRS, Laboratoire de Physique, F-69342 Lyon, France

Numerical computation of the texture of cholesteric droplets

Theory

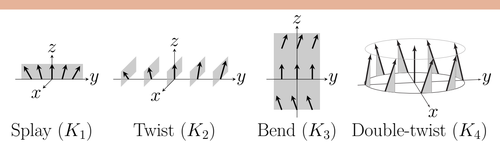


cholesteric phase:

- orientational order with spontaneous twist
- no positional order

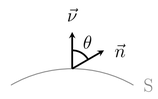
Unit director field: $\vec{n}_s = \underset{\vec{n},\ |\vec{n}|=1}{\operatorname{argmin}} F[\vec{n}]$, with $F[\vec{n}] = F_f[\vec{n}] + F_e[\vec{n}] + F_s[\vec{n}]$

Frank elastic energy F_f:



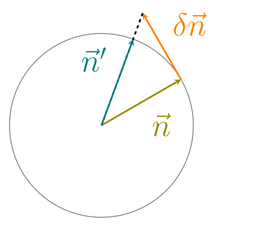
Electric energy F_e: $-\epsilon_0\epsilon_a(\vec{n}\cdot\vec{E})^2$

Surface energy F_s: $\gamma(\theta) \sim W_a(\theta-\theta_a)^2$ when $|\theta-\theta_a| \ll 1$



Numerical minimization method[1]

- Discretization with $\mathcal{Q}_1$ finite elements: $F[\vec{n}] \to f(\boldsymbol{N})$ with $\boldsymbol{N} = \begin{pmatrix} \vec{n}_1 \\ \vdots \\ \vec{n}_M \end{pmatrix}$
- Iterative minimization:
  - $\star\ \boldsymbol{N}_{(k)}$ verifying $\vec{n}_\beta \cdot \vec{n}_\beta = 1$
  - $\star\ \boldsymbol{N}_{(k+1)} = \mathcal{P}\left(\boldsymbol{N}_{(k)} + \boldsymbol{\delta N}\right)$, where $\mathcal{P}$ is the normalization operation $\vec{n}_\beta \to \vec{n}_\beta / |\vec{n}_\beta|$
  - $\star\ \boldsymbol{\delta N}$ found with the truncated conjugate gradient algorithm (trust region strategy)

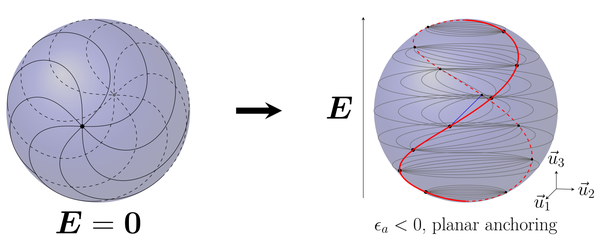


Essential properties:

- $f\left(\boldsymbol{N}_{(k+1)}\right) < f\left(\boldsymbol{N}_{(k)}\right)$: the energy always decreases
- Quadratic convergence near the minimum

Measurement of the anchoring energy at the Chol-I interface of twisted bipolar droplets

Structure under electric field

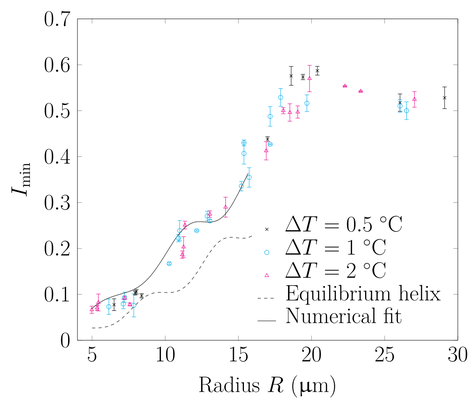


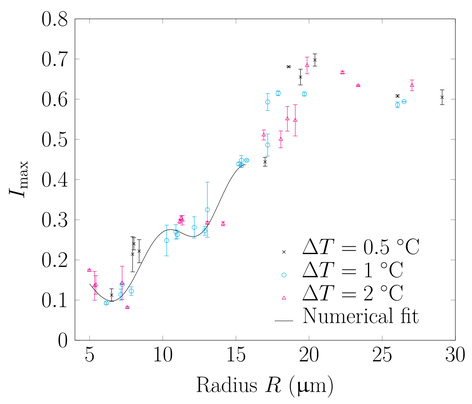
Competition between the energy of the disclination lines and the bulk twist energy:

- Partially unwound helix
- Effect highly dependent on R/l_a, with $l_a = K_1/W_a$

Optical transmission at the droplet center

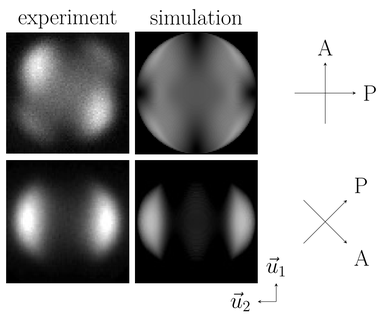
- Intensity under crossed polarizers $\vec{p}$ and $\vec{a}$: $I(\varphi)$, where $\varphi = \text{angle}\,(\vec{u}_1, \vec{p})$
- Because of the Lehmann effect, the texture rotates at angular velocity $\omega \propto \Delta T$
- Only the minimum and the maximum of $I(\varphi)$ are kept





- **Spherical shape** as long as $2R$ is smaller than the sample thickness
- Numerical fit gives $l_a \approx 0.8$ µm for the mixture CCN-37 + 0.527 % R811

Simulated and experimental images

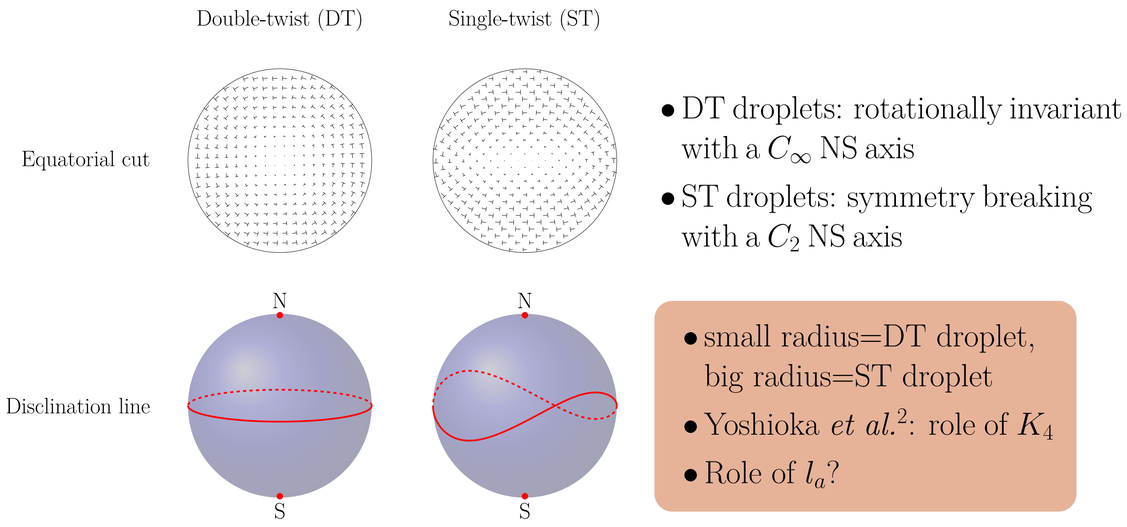


- Experimental images: Köhler illumination setup
- Simulated images: Jones matrix method

Rather good but not perfect agreement.
Role of the light deviation effect?

Stability analysis of banded and double-twist droplets

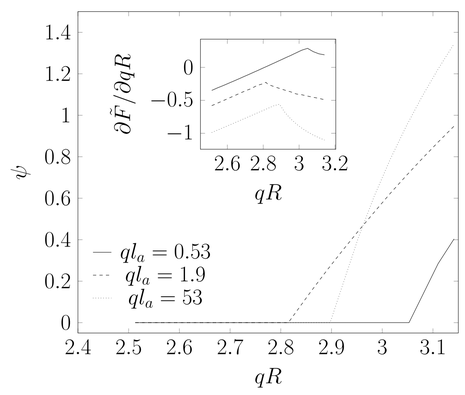
Structure of the droplets



- DT droplets: rotationally invariant with a C_∞ NS axis
- ST droplets: symmetry breaking with a C_2 NS axis

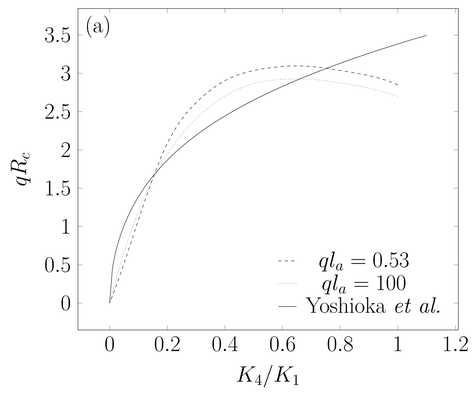
- small radius=DT droplet, big radius=ST droplet
- Yoshioka *et al.*[2]: role of K_4
- Role of l_a?

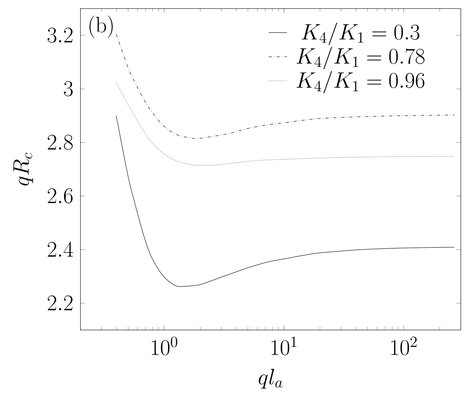
Numerical study of the transition

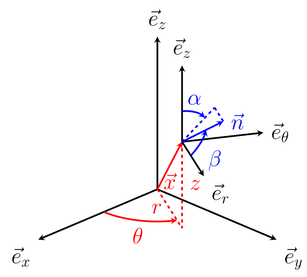


Order parameter $\psi = \frac{1}{V}\int_V \mathrm{d}V \left[\frac{\mathrm{D}\vec{n}}{\mathrm{D}\Theta}\right]^2$, where $\mathrm{D}/\mathrm{D}\Theta$ is the variation rate associated with a solid rotation around the NS axis

- Transition of the second order
- The critical radius R_c depends on K_4 and l_a







$$\begin{aligned}\vec{n}(r,z) &= \cos\beta(r,z)\vec{e}_r \\ &- \sin\beta(r,z)\sin\alpha(r,z)\vec{e}_\theta \\ &+ \sin\beta(r,z)\cos\alpha(r,z)\vec{e}_z\end{aligned}$$

- Yoshioka *et al.* ansatz[2]:

$$\alpha(r,z) = q_{\text{eff}} r$$
$$\beta(r,z) = \pi/2$$

- Numerical profiles very different from this ansatz

References

[1] G. Poy, F. Bunel, and P. Oswald, Phys. Rev. E. (to be published)
[2] J. Yoshioka, F. Ito, Y. Tabe, Soft Matter, **12**, 2400 (2016)