**Texture of cholesteric droplets: finite element simulations and experiments**

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### Numerical computation of the texture of cholesteric droplets

#### Theory

<table>
<thead>
<tr>
<th>$T_{coexistence}$</th>
<th>$T_{cholesteric}$</th>
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<tbody>
<tr>
<td>cholesteric phase</td>
<td>cholesteric, isotropic phase</td>
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</table>

- **Unit director field**: $\vec{r}_i = \text{argmin}_{\vec{r}_i} \, F[\vec{r}]$, with $F[\vec{r}] = F_T[\vec{r}] + F_S[\vec{r}] + F_P[\vec{r}]$

- **Frank elastic energy** $F_T$: $\Psi_{\vec{r}_i} (K_i) - \Psi_{\vec{r}_i} (K_i)$ for $\vec{r}_i = \vec{n}_i$

- **Electric energy** $F_E$: $-\vec{e} \cdot \vec{E}$

- **Surface energy** $F_S$: $\gamma(\theta) - W_f(\theta; \gamma)$ when $\theta < 1$

#### Numerical minimization method

- **Discretization** with $Q_i$: finite elements: $F(\vec{r}) \rightarrow f(\vec{N})$ with $\vec{N} = \begin{pmatrix} \vec{n}_i \\ \vec{r}_i \end{pmatrix}$

- **Iterative minimization**:
  - $N_{(i+1)}$ verifying $\vec{r}_i \cdot \vec{r}_i = 1$
  - $N_{(i+1)} = P (N_{(i)} + \Delta N)$ where $P$ is the normalization operation $\vec{r}_i \rightarrow \vec{r}_i / |\vec{r}_i|$

- **$\Delta N$** found with the truncated conjugate gradient algorithm (trust region strategy)

#### Essential properties:

- $f(N_{(i+1)}) < f(N_{(i)})$: the energy always decreases
- Quadratic convergence near the minimum

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### Measurement of the anchoring energy at the Chol-I interface of twisted bipolar droplets

#### Structure under electric field

- Competition between the energy of the disclination lines and the bulk twist energy:
  - Partially unwound helix
  - Effect highly dependent on $R/\alpha$ with $\alpha = K_f/W_a$

- **Optical transmission at the droplet center**
  - Intensity under crossed polarizers $\bar{p}$ and $\bar{q}$: $I(\phi)$, where $\phi = \text{angle}(\bar{a}_1, \bar{p})$
  - Because of the Lehmann effect, the texture rotates at angular velocity $\omega = \Delta T$
  - Only the minimum and the maximum of $I(\phi)$ are kept

- **Spherical shape** as long as $2R$ is smaller than the sample thickness
- **Numerical fit** $l_a \approx 0.8 \ \mu m$ for the mixture CCN-37 + 0.527 % R811

#### Simulated and experimental images

- Experimental images: Köhler illumination setup
- Simulated images: Jones matrix method

- Rather good but not perfect agreement. Role of the light deviation effect?

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### Stability analysis of banded and double-twist droplets

#### Structure of the droplets

- **DT droplets**: rotationally invariant with a $C_\infty NS$ axis
- **ST droplets**: symmetry breaking with a $C_3 NS$ axis

- **Small radius**: DT droplet, big radius: ST droplet
- Yoshieka et al. $^2$: role of $K_4$
- Role of $l_a$?

#### Numerical study of the transition

- Order parameter $\psi = \frac{1}{2} \int dV \left[ \frac{D\Theta}{DO} \right]^2$ where $D/D_0$ is the variation rate associated with a solid rotation around the $NS$ axis

- **Transition of the second order**
- The critical radius $R_c$ depends on $K_4$ and $l_a$

- Yoshieka et al. ansatz $^2$: $\alpha(r, z) = q(\alpha r + z)\Theta$
  - $\beta(r, z) = \pi/2$

- **Numerical profiles very different from this ansatz**

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### References

1. G. Poy, F. Bunel, and P. Oswald, Phys. Rev. E. (to be published)