Lehmann effect:
The end of the Leslie paradigm

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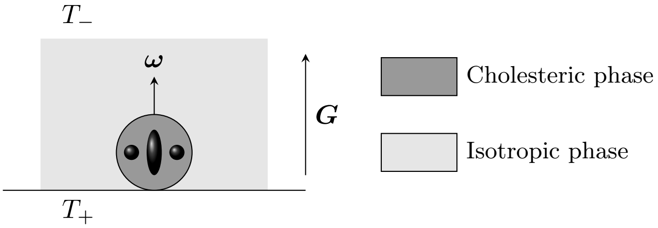
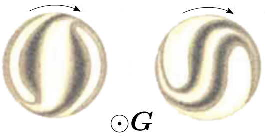
August 1, 2016

Plan

First observations by Lehmann





Lehmann, 1900:

- coexistence of cholesteric droplets with the isotropic fluid
- rotation of the droplets internal texture when heated from below

O. Lehmann. *Ann. Phys.*, 307(8):649–705, 1900

Leslie interpretation of the Lehmann experiment

First explanation by Leslie in 1968:

- nematic phase: symmetry $D_{\infty h} \Rightarrow$ invariant by inversion

F. M. Leslie. *Proc. R. Soc. A*, 307(1490):359–372, 1968

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  $\Rightarrow$ existence of a torque on the director: $\mathbf{\Gamma}_L = \nu\, \boldsymbol{n} \times [\boldsymbol{n} \times \boldsymbol{G}]$

  ν: Leslie thermomechanical coefficient

  $\boldsymbol{n}$: director

  $\boldsymbol{G}$: temperature gradient

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Leslie paradigm

The rotation of the texture in the Lehmann experiment is due to the Leslie thermomechanical torque $\mathbf{\Gamma}_L$

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Plan

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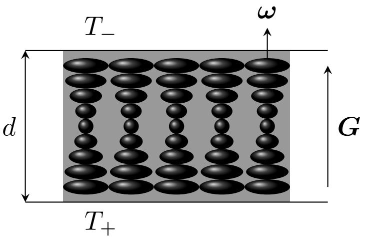
Measurement of the Leslie thermomechanical constant



- cholesteric sample with sliding planar anchoring

crossed polarizers

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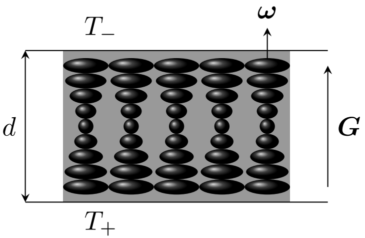

- cholesteric sample with sliding planar anchoring
- solution of the torque equation:

$$\omega = -\frac{\nu\, G}{\gamma_1 + 2\gamma_s/d}$$

γ_1: bulk rotational viscosity

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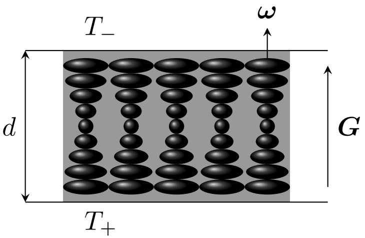

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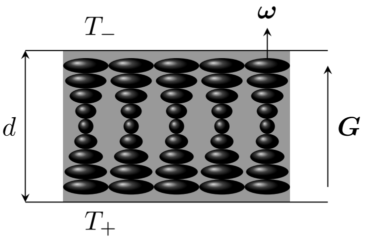

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- sign(ω) given by the sense of rotation of the negative defects

crossed polarizers

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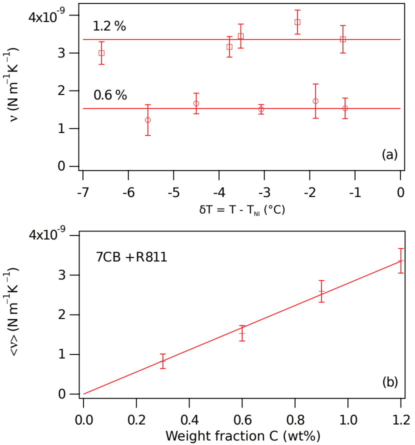
Results



- ν independent of the temperature T and proportional to C

P. Oswald. *EPL*, 108(3):36001, 2014

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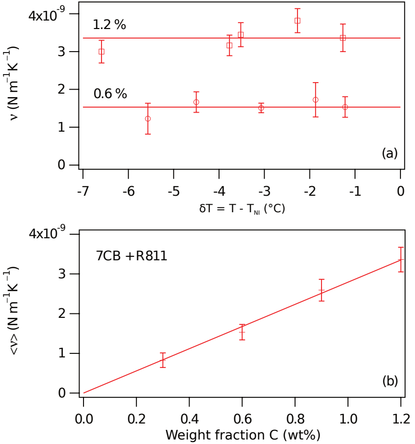
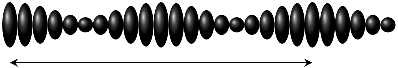


- ν independent of the temperature T and proportional to C
- spontaneous twist $q = 2\pi/P$ also proportional to C



P: cholesteric pitch

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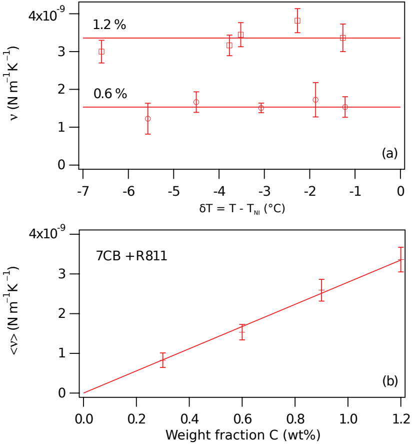

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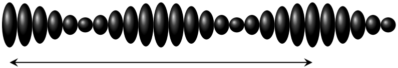

P: cholesteric pitch

LC	7CB	
Dopant	R811	CC
q	$+$	$-$
ν	$+$	$+$
$R = \frac{\nu}{q}$ (fN/K)	3.6	-4.2

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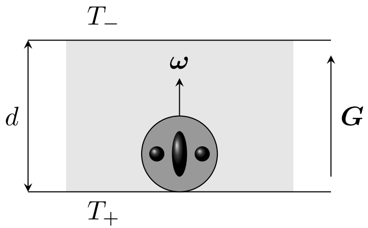
Thermomechanical model for the Lehmann effect



- Droplets with a banded texture in coexistence with the isotropic fluid

natural light

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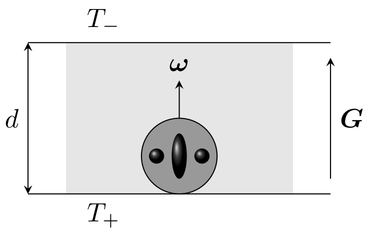

- Droplets with a banded texture in coexistence with the isotropic fluid
- Leslie thermomechanical model without backflow:

$$-\frac{\nu G}{\gamma_1 \omega} = 1 + I[\boldsymbol{n}]$$

with $I[\boldsymbol{n}] \underset{R\to 0}{\longrightarrow} 0$

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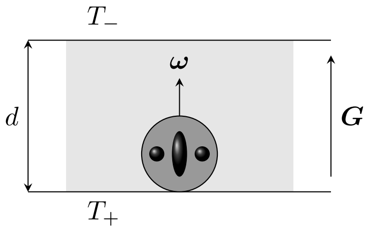

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with $I[\boldsymbol{n}] \xrightarrow[R \to 0]{} 0$

- To test this model, we describe our data with a similar relation:

$$-\frac{\bar{\nu} G}{\gamma_1 \omega} = 1 + f(qR)$$

and compare the measured values of $\bar{\nu}$ and ν

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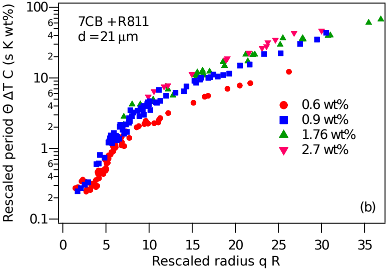

$$-\frac{\bar{\nu}G}{\gamma_1\omega} = 1 + f(qR)$$

The extrapolation to zero of the period curves gives $\bar{\nu}$ up to a known multiplicative factor

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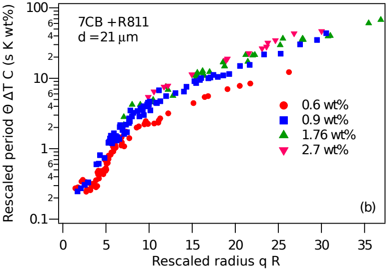

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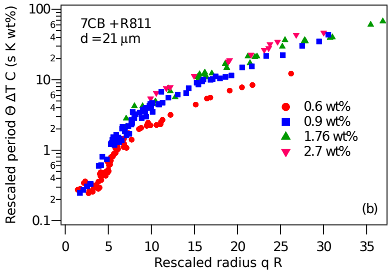

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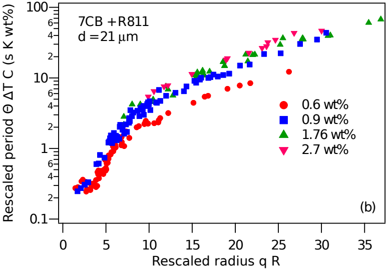

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LC	7CB	
Dopant	R811	CC
$R = \frac{\nu}{q}$ (fN/K)	3.6	-4.2
$\bar{R} = \frac{\bar{\nu}}{q}$ (fN/K)	$4.6\ 10^3$	

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Summary

Leslie effect:

- ν/q dependant of the chiral dopant and positive or negative

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Lehmann effect due to the chirality of the director field and/or the chirality of the phase?

New problematic

Can we observe the Lehmann effect in droplets of an **achiral phase** with a **chiral director field**?

Plan

Plan

How to obtain twist

Frank-Oseen elastic energy:

$$F[\boldsymbol{n}] = \int_V \frac{\mathrm{d}V}{2} \left(K_1 \left[\nabla \cdot \boldsymbol{n}\right]^2 + K_2 \left[\boldsymbol{n} \cdot \nabla \times \boldsymbol{n}\right]^2 + K_3 \left[\boldsymbol{n} \times \nabla \times \boldsymbol{n}\right]^2 \right)$$

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Two possible origins for a twisted director field:

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- action of a topological constraint on the LC domain surface:
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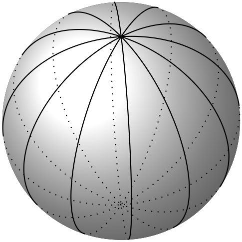
  - no need for a chiral phase

Stability of bipolar configuration

Topological constraint:
planar anchoring



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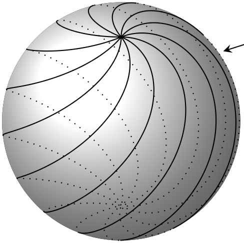
twist $\sim$ splay, bend



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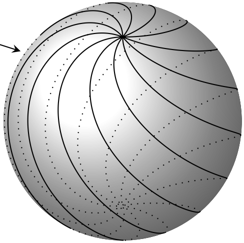

R. D. Williams. *J. Phys. A*, 19(16):3211, 1986

Plan

Rotation of twisted bipolar droplets

- Lyotropic chromonic nematic used: water + 30% SSY ($K_2/K_1 \simeq 0.16$, $K_2/K_3 \simeq 0.12$)

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Rotation only due to the twist of the director field

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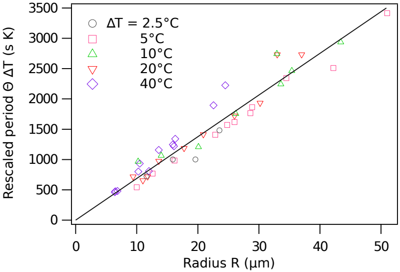
Relation between the period and the radius



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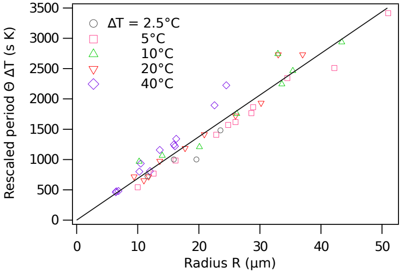

- Angular velocity $\omega = 2\pi/\Theta$ proportional to $\boldsymbol{G}$
- Period Θ proportional to the radius R

Plan

Thermomechanical coupling of Akopyan and Zel'dovich

- Akopyan/Zel'dovich thermomechanical torque $\mathbf{\Gamma}_{\text{nem}} = \boldsymbol{n} \times \boldsymbol{f}_{\text{nem}}$ on the director $\boldsymbol{n}$ of a nematic phase, with:

$$\begin{aligned} \boldsymbol{f}_{\text{nem}} &= \bar{\xi}_1 \ (\nabla \cdot \boldsymbol{n}) \ \boldsymbol{G} \\ &+ \bar{\xi}_2 \ ([\nabla \times \boldsymbol{n}] \cdot \boldsymbol{n}) \ (\boldsymbol{n} \times \boldsymbol{G}) \\ &+ \bar{\xi}_3 \ ([\nabla \times \boldsymbol{n}] \times \boldsymbol{n}) \ (\boldsymbol{n} \cdot \boldsymbol{G}) \,, \end{aligned}$$

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- Theoretical prediction without backflow:

$$-\frac{\gamma_1 \; \omega \; R}{G} = \bar{\xi}_1 \; I_1[\boldsymbol{n}] + \bar{\xi}_2 \; I_2[\boldsymbol{n}] + \bar{\xi}_3 \; I_3[\boldsymbol{n}]$$

$I_i[\boldsymbol{n}]$: rescaled functionals of the texture on the unit sphere

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Finite-Element simulation of the texture

- Droplet texture given by the minimum of:

$$\mathcal{L}[\boldsymbol{n}, \lambda] = F[\boldsymbol{n}] + \int_S \mathrm{d}S \, \frac{W_a}{2} \, (\boldsymbol{n} \cdot \boldsymbol{\nu})^2 + \int_V \mathrm{d}V \, \lambda \, (\boldsymbol{n} \cdot \boldsymbol{n} - 1)$$

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- Non-linear problem ⇒ Newton-Raphson system projected on a FE space:

$$(\boldsymbol{n},\lambda) \to (\boldsymbol{n},\lambda) + \alpha\, (\boldsymbol{\delta n}, \delta\lambda),\ \alpha \in]0,1[$$

$$\begin{pmatrix} H_{\boldsymbol{nn}} & H_{\lambda\boldsymbol{n}} \\ H_{\boldsymbol{n}\lambda} & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\delta N} \\ \boldsymbol{\delta\Lambda} \end{pmatrix} = -\begin{pmatrix} \boldsymbol{D_n} \\ \boldsymbol{D_\lambda} \end{pmatrix}$$

Finite-Element simulation of the texture

- Droplet texture given by the minimum of:

$$\mathcal{L}[\boldsymbol{n},\lambda] = F[\boldsymbol{n}] + \int_S \mathrm{d}S\, \frac{W_a}{2}\, (\boldsymbol{n}\cdot\boldsymbol{\nu})^2 + \int_V \mathrm{d}V\, \lambda\, (\boldsymbol{n}\cdot\boldsymbol{n} - 1)$$

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- After convergence, the solution $\boldsymbol{n}^*$ depends only on three parameters: (K_2/K_1), (K_3/K_1) and $(R/l_a) = (R\,W_a)/K_1$

Numerical results

$$-\frac{\gamma_1\ \omega\ R}{G} = \bar{\xi}_1\ I_1[\boldsymbol{n}] + \bar{\xi}_2\ I_2[\boldsymbol{n}] + \bar{\xi}_3\ I_3[\boldsymbol{n}]$$

- (K_2/K_1) and (K_2/K_3) known
  $\Rightarrow I_i$ depend only on (R/l_a)

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$$\Theta\ \Delta T = \frac{2\pi\gamma_1}{\bar{\xi}\ a}\ R\ J\left(\frac{R}{l_a}\right)$$

$J(R/l_a)$ is computed with our FE code

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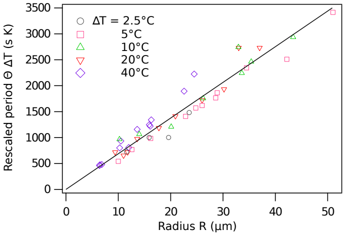

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  $\Rightarrow J(R/l_a)$ constant
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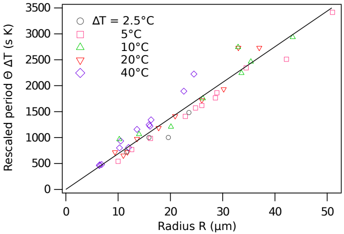

- Θ linear in R
  $\Rightarrow J(R/l_a)$ constant
  $\Rightarrow$ Strong anchoring
- With $J(R/l_a) \simeq J(\infty)$, we find $\bar{\xi} = 76$ pN/K

Good qualitative agreement; Quantitative agreement?

Plan

Conclusion

- Lehmann effect in an achiral phase with a twisted director field:

The Lehmann effect is only due to the chirality of the director field

$\Downarrow$

The Leslie thermomechanical model cannot explain alone the Lehmann effect

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- Good qualitative agreement with the Akopyan/Zel'dovich thermomechanical model
- **But** large value of $\bar{\xi}$ in comparison with the theoretical prediction of Akopyan and Zel'dovich
- Question: quantitative agreement with the value of $\bar{\xi}$ below T_{NI}?

Thank you for your attention!

Equivalent expressions for the thermomechanical force

Dequidt convention:

$$\boldsymbol{f}_{TM} = \bar{\xi}_1\ (\boldsymbol{\nabla}\cdot\boldsymbol{n})\,\boldsymbol{G} + \bar{\xi}_2\ (\boldsymbol{n}\cdot[\boldsymbol{\nabla}\times\boldsymbol{n}])\,(\boldsymbol{n}\times\boldsymbol{G}) + \bar{\xi}_3\ (\boldsymbol{n}\cdot\boldsymbol{G})\,([\boldsymbol{\nabla}\times\boldsymbol{n}]\times\boldsymbol{n}) - \bar{\xi}_4\,\boldsymbol{\nabla}\cdot(\boldsymbol{G}\otimes\boldsymbol{n} - [\boldsymbol{G}\cdot\boldsymbol{n}]\ \mathbb{I})$$

Akopyan/Zel'dovich convention:

$$\boldsymbol{f}_{TM} = (-\xi_1 + \xi_3/2)\,(\boldsymbol{\nabla}\cdot\boldsymbol{n})\ \boldsymbol{G} + \xi_2\ (\boldsymbol{n}\cdot\boldsymbol{\nabla}\times\boldsymbol{n})\,\boldsymbol{n}\times\boldsymbol{G} + (\xi_3/2 - \xi_4/2)\ (\boldsymbol{n}\cdot\boldsymbol{G})\ ([\boldsymbol{\nabla}\times\boldsymbol{n}]\times\boldsymbol{n}) - (\xi_3/2)\,([\boldsymbol{\nabla}\boldsymbol{n}]\cdot\boldsymbol{G} + [\boldsymbol{G}\cdot\boldsymbol{\nabla}]\boldsymbol{n})$$

Brandt/Pleiner convention:

$$\boldsymbol{f}_{TM} = -\gamma_1\pi_1\,[\boldsymbol{\nabla}\cdot\boldsymbol{n}]\,\boldsymbol{G} - \gamma_1\pi_2\,[\boldsymbol{\nabla}\boldsymbol{n}]\cdot\boldsymbol{G} - \gamma_1\pi_3\,[\boldsymbol{G}\cdot\boldsymbol{\nabla}]\boldsymbol{n} - \gamma_1[\pi_4 - \pi_3]\,[\boldsymbol{n}\cdot\boldsymbol{G}]\,[\boldsymbol{n}\cdot\boldsymbol{\nabla}]\boldsymbol{n}$$

Correspondence between conventions:

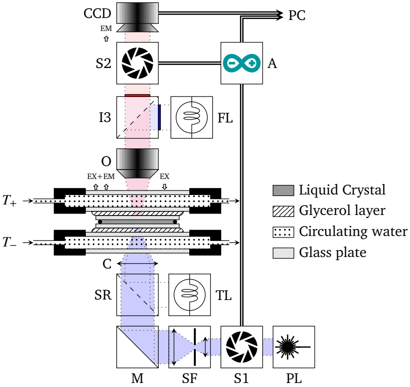
$$\bar{\xi}_1 = -\gamma_1(\pi_1+\pi_2+\pi_3)$$
$$\bar{\xi}_2 = -\gamma_1\pi_3$$
$$\bar{\xi}_3 = -\gamma_1\pi_4$$
$$\bar{\xi}_4 = -\gamma_1(\pi_2+\pi_3)$$

$$\bar{\xi}_1 = -\xi_1 - \xi_3/2$$
$$\bar{\xi}_2 = \xi_2 - \xi_3/2$$
$$\bar{\xi}_3 = -\xi_4/2$$
$$\bar{\xi}_4 = -\xi_3$$

H. Pleiner and H. R. Brand. Springer, 1996

A. Dequidt, G. Poy, and P. Oswald. *Soft Matter*, 2016, to be published

Photobleaching experiment



- LC mixture doped with fluorescent molecules
- Gaussian beam of a laser focalized near a rotating droplet
- The bleached spot is not advected ⇒ droplet not rotating as a solid

G. Poy and P. Oswald. *Soft Matter*, 12(9):2604–2611, 2016