Guiding principles of light in frustrated chiral birefringent systems

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1 Introduction

2 Light propagation in anisotropic media

3 Role of chirality in the non-linear response of a confined cholesteric

4 Interaction between light and topological solitons

5 Summary
Chirality in everyday life

- Chiral object: distinguishable from its mirror image.
- A common example: propeller.

Without chirality, this conversion is not possible.
The cholesteric phase: a chiral anisotropic soft material

- Nematic liquid crystal: no positional order, mean molecular orientation $\mathbf{n}$
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- Effect of chirality: helix structure for the director vector field $n$. 
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Frustration: confining cholesterics between two plates

- Surface constraint: molecules must be normal to the confining surface

P. J. Ackerman et al., *Scientific Reports* 2 (2012)
Frustration: confining cholesterics between two plates

- Surface constraint: molecules must be normal to the confining surface

- Arbitrary shapes can be written with thread-like structures!

P. J. Ackerman et al., *Scientific Reports* 2 (2012)
Frustration: confining cholesterics inside droplets

Topological zoo of free standing knots

Lasing in a cholesteric droplet: an omnidirectional microscopic coherent light source

M. Humar, *Liquid Crystals* 43 (2016)
Non-linear optical response of liquid crystal systems:

Problematic 1

Role of chirality in the non-linear optical response of a frustrated cholesteric?

Two classes of soft chiral topological solitons in frustrated cholesterics

Problematic 2
Localized and robust chiral birefringent structures ⇒ interesting interaction with light?
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Motivations

- Recent advances in LC-based light application: tunable microresonators, micro-optical elements, diffraction gratings...
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- Simulation tools for light propagation:
  - Jones method (fast but inaccurate, easy to code)
  - Finite Difference Time Domain (accurate but slow, open-source, complex to use)
  - Other methods (in-house implementation)
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Need for advanced light propagation code, if possible open-source
First approach: Hamiltonian ray-tracing and energy transport

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\[ \mathcal{F}^{(\alpha)} = n_{\text{eff}} \sqrt{q} E \] conserved along a ray

Advantage: intuitive physical interpretation

Light propagation in anisotropic media

Second approach: physics-based splitting of the wave equation

- Wave-equation in anisotropic media: \[ \partial_k \partial_k \delta_{ij} - \partial_i \partial_j + k_0^2 \epsilon_{ij} \] \( E_j = 0 \)

G. Poy and S. Žumer, *Optics Express* **28** (2020)
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  \[ i \partial_z E_\perp = -\mathcal{P} E_\perp \]

- What’s inside \( \mathcal{P} \)?

\[ \text{Phase op. } K \sim k_0^2 \epsilon \]
\[ \text{Walkoff op. } W \sim (\epsilon u_z) \otimes \nabla_\perp \]
\[ \text{Diffraction op. } D \sim \Delta_\perp \]

Advantage: fast and accurate simulations

G. Poy and S. Žumer, *Optics Express* 28 (2020)
Nemaktis: an open-source package for polarised microscopy

- The open-source package includes:
  - Low-level simulation backends (C++, python)
  - An easy-to-use high-level interface (python)
  - A graphical interface for micrographs simulation

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B. Berteloot et al., *Soft Matter* 16 (2020)
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- Closed-source BPM code for advanced uses: wide-angle beam deflection, non-linear optics, etc.

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Spatial light solitons in liquid crystals: nematicons

Motivations

Studied systems in the past 20 years:

- Thick samples with planar $n$
- Thick samples with cholesteric helix
- Thin samples with homeotropic $n$

What about confined chiral systems? Can we amplify the optical response with chirality?
Orientational elasticity and non-linear interactions

Free energy of the liquid crystal phase:

$$F[n, E] = \int_V dV \left[ f_F(n, \nabla n) - \frac{\epsilon_0 \epsilon_a |n \cdot E|^2}{4} \right]$$
Orientational elasticity and non-linear interactions

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Non-linear iterative scheme:

- \( E_{k+1} \): BPM solution with \( \epsilon = \epsilon_{\perp} I + \epsilon_a n_k \otimes n_k \)
- \( n_{k+1} = n_k + \mu \frac{\delta F}{\delta n} [n_k, E_{k+1}] \)
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Typical running time for a mesh of \( 3 \times 10^6 \) points: \( 4 \text{ s / step} \)
(Full resolution of Maxwell equations for the same mesh: \( \sim 1 \text{ h} \))
Role of chirality in the non-linear response of a confined cholesteric

Side-view observations

Side slice of beam intensity (simulation):

Side slice of 3PF signal (experiment):
Top-view observations

Top view of the thickness-averaged laser intensity (simulation):

- Linear optical regime
- Non-linear optical regime

Top view of the scattered laser light (experiments):

- Linear optical regime
- Non-linear optical regime
Top-view observations

Top view polarised optical micrograph:

Simulation

Experiment

Side slice of director field

Mid-sample slice of director field
Role of chirality in the non-linear response of a confined cholesteric

Chirality-enhanced non-linear optical response

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Potential for low-power non-linear optical photonics devices (e.g. active lenses)

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Transmission and/or reflection with line-like structure

Reflection of incident extraordinary beam ($\theta_i = 70^\circ$):
Transmission and/or reflection with line-like structure

Transmission of incident extraordinary beam ($\theta_i = 55^\circ$):
Interaction between light and topological solitons

Description with a generalization of Snell’s law

From an exact eigenmode decomposition of Maxwell equations:

\[ n^{(\alpha,m)} \sin \theta^{(\alpha,m)} = n_i \sin \theta_i \]
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- In our system, \( n^{(\alpha,m)} \) = effective index of eigenmode \( \{\alpha, m\} \) far from the soliton
  - \( \alpha = e, o \): polarisation state
  - \( m = 1, 2, \ldots \): mode index
Interaction between light and topological solitons

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\[ \theta^{(\alpha,m)} \] does not depend on the choice of topological soliton!
(but Fresnel coefficients do)
Interaction between light and topological solitons

Comparison with experiments

Small mode index approximation in thick samples:

\[ n^{(\alpha,m)} \approx n_\alpha \sqrt{1 - \left(\frac{m}{m_0}\right)^2} \approx n_\alpha \]
Comparison with experiments

Splitting of eigenmode packets (strongly depends on x-profile): $n^{(\alpha,m_1)} \neq n^{(\alpha,m_2)}$
Interaction with point-like topological solitons

Simplification with 2D rays: \( dp_y/dz \approx -(\epsilon_a/2n_0) g \), where \( g \equiv \partial n_z^2/\partial y \)
Light deflection and lensing with pinned torons

A. Hess et al., Physical Review X 10 (2020)
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Chirality and soft topological solitons unlocks new possibilities to control the flow of light at the microscopic level
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- Chirality-enhanced optical response: towards enriched opto-mechanical interactions
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- Chirality-enhanced optical response: towards enriched opto-mechanical interactions
- Matter transforming light, light transforming matter: what happens when we combine everything?
Take-home message

Chirality and soft topological solitons unlocks new possibilities to control the flow of light at the microscopic level

- Chirality-enhanced optical response: towards enriched opto-mechanical interactions
- Matter transforming light, light transforming matter: what happens when we combine everything?

Next step: establishment of a general chirality-enhanced topological optomechanics framework.