

Guiding principles of light in frustrated chiral birefringent systems

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JAVNA AGENCIJA ZA RAZISKOVALNO DEJAVNOST
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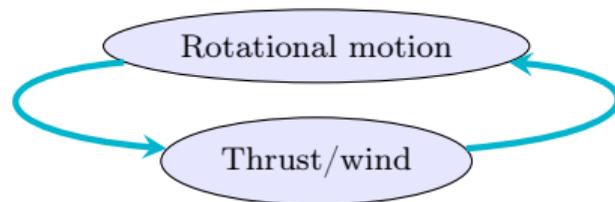


Outline

- 1 Introduction
- 2 Light propagation in anisotropic media
- 3 Role of chirality in the non-linear response of a confined cholesteric
- 4 Interaction between light and topological solitons
- 5 Summary

Chirality in everyday life

- Chiral object: distinguishable from its mirror image.
- A common example: propeller.



- Without chirality, this conversion is not possible.

The cholesteric phase: a chiral anisotropic soft material

- Nematic liquid crystal: no positional order, mean molecular orientation \mathbf{n}

The cholesteric phase: a chiral anisotropic soft material

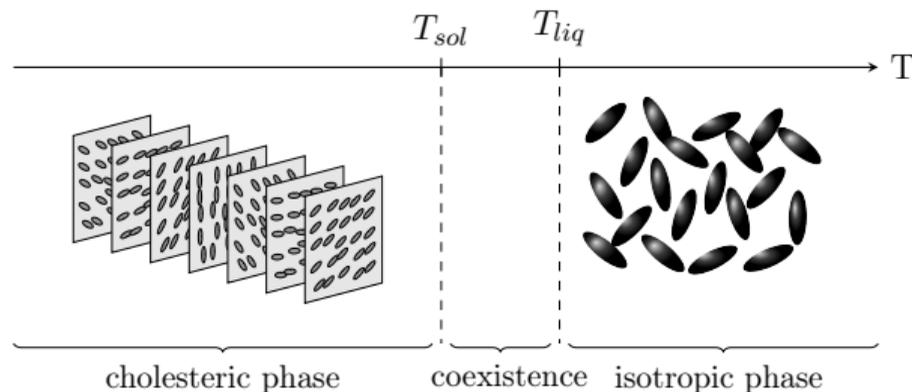
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- Nematic phase + chiral molecules: cholesteric phase.

The cholesteric phase: a chiral anisotropic soft material

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- Effect of chirality: helix structure for the director vector field \mathbf{n} .

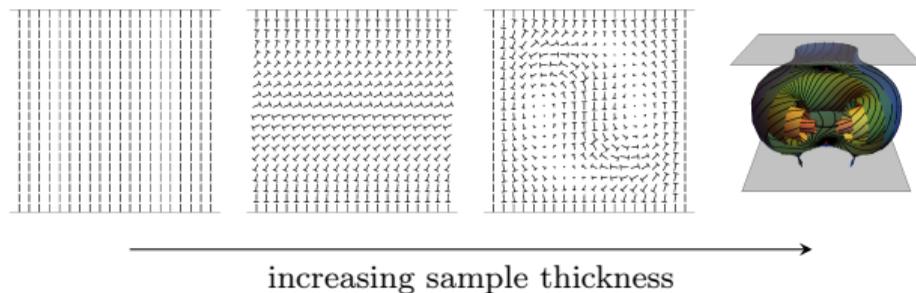
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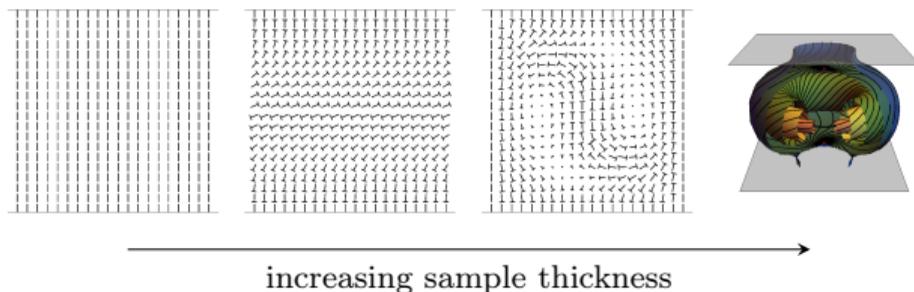
Frustration: confining cholesterics between two plates

- Surface constraint: molecules must be normal to the confining surface

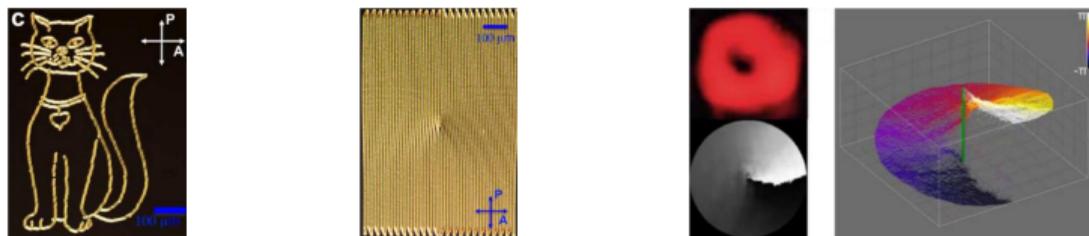


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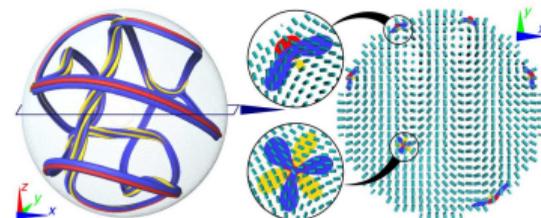
- Arbitrary shapes can be written with thread-like structures!



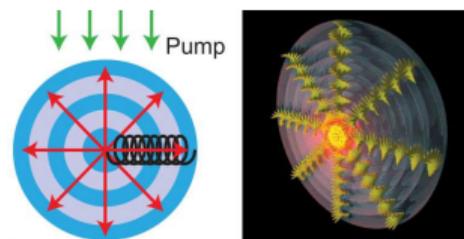
P. J. Ackerman et al., *Scientific Reports* **2** (2012)

Frustration: confining cholesterics inside droplets

Topological zoo of free standing knots



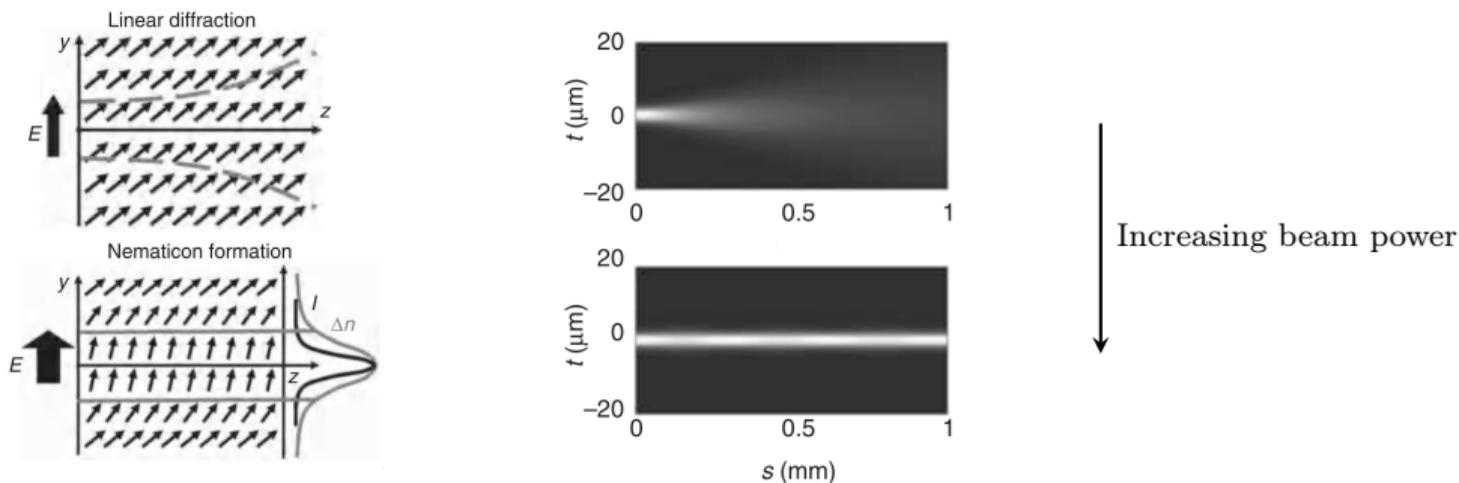
Lasing in a cholesteric droplet: an omnidirectional microscopic coherent light source



D. Seč, S. Čopar and S. Žumer, *Nature communications* **5** (2014)
 M. Humar, *Liquid Crystals* **43** (2016)

Problematics

Non-linear optical response of liquid crystal systems:



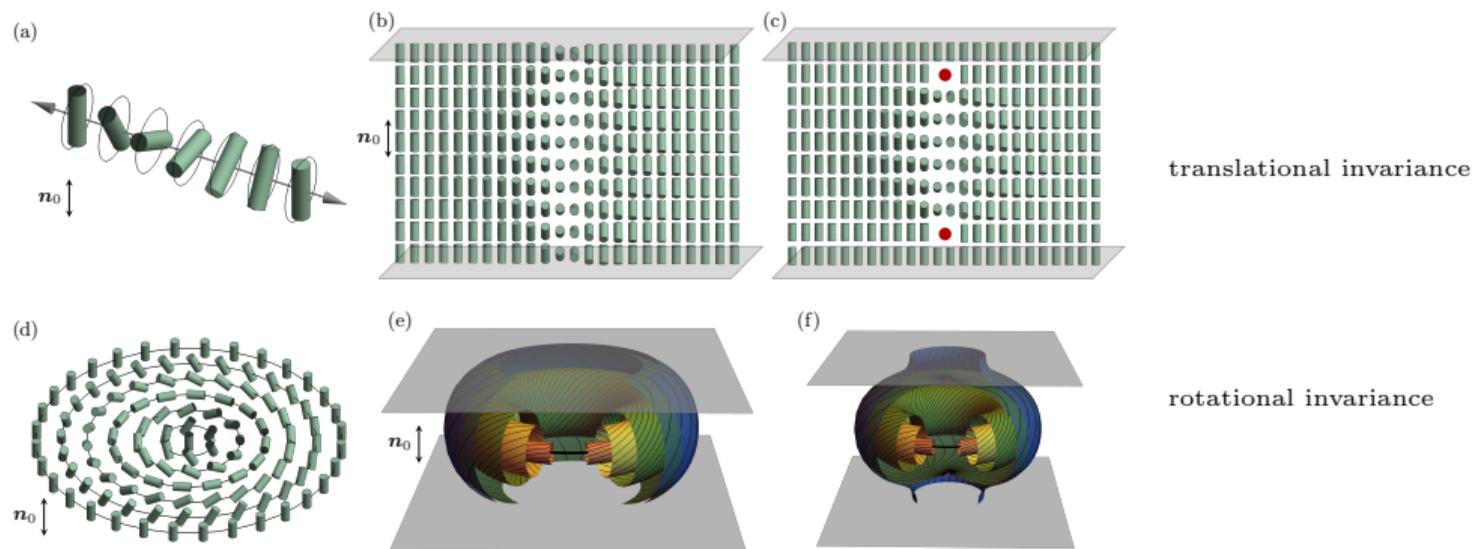
G. Assanto. *Nematicons*. John Wiley & Sons, 2013

Problematic 1

Role of chirality in the non-linear optical response of a frustrated cholesteric?

Problematics

Two classes of soft chiral topological solitons in frustrated cholesterics



Problematic 2

Localized and robust chiral birefringent structures \Rightarrow interesting interaction with light?

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Motivations

- Recent advances in LC-based light application: tunable microresonators, micro-optical elements, diffraction gratings...

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- Simulation tools for light propagation:
 - ★ Jones method (fast but inaccurate, easy to code)
 - ★ Finite Difference Time Domain (accurate but slow, open-source, complex to use)
 - ★ Other methods (in-house implementation)

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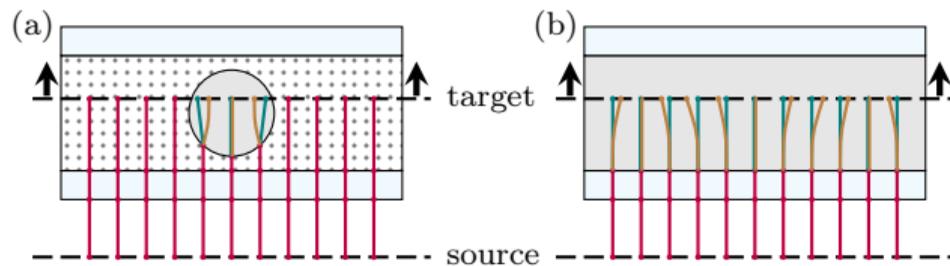
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Need for advanced light propagation code, if possible open-source

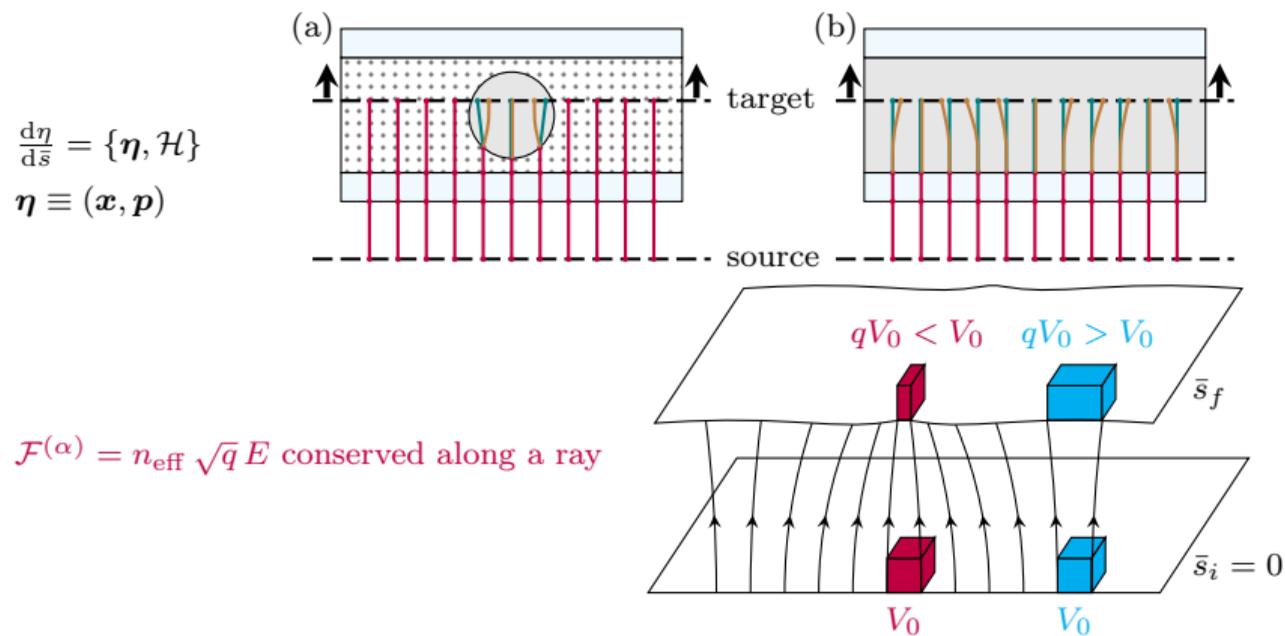
First approach: Hamiltonian ray-tracing and energy transport

$$\frac{d\eta}{ds} = \{\eta, \mathcal{H}\}$$

$$\eta \equiv (\mathbf{x}, \mathbf{p})$$



First approach: Hamiltonian ray-tracing and energy transport



Advantage: intuitive physical interpretation

Second approach: physics-based splitting of the wave equation

- Wave-equation in anisotropic media: $[\partial_k \partial_k \delta_{ij} - \partial_i \partial_j + k_0^2 \epsilon_{ij}] E_j = 0$

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- Wave-equation in anisotropic media: $[\partial_k \partial_k \delta_{ij} - \partial_i \partial_j + k_0^2 \epsilon_{ij}] E_j = 0$
- After eliminating E_z and keeping only forward modes:

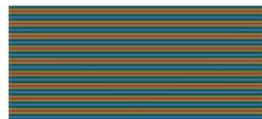
$$i\partial_z \mathbf{E}_\perp = -\mathcal{P} \mathbf{E}_\perp$$

Second approach: physics-based splitting of the wave equation

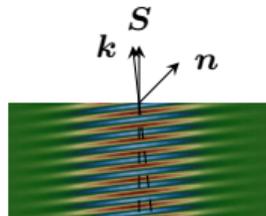
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- What's inside \mathcal{P} ?



Phase op. $K \sim k_0^2 \epsilon$



Walkoff op. $W \sim (\epsilon u_z) \otimes \nabla_\perp$

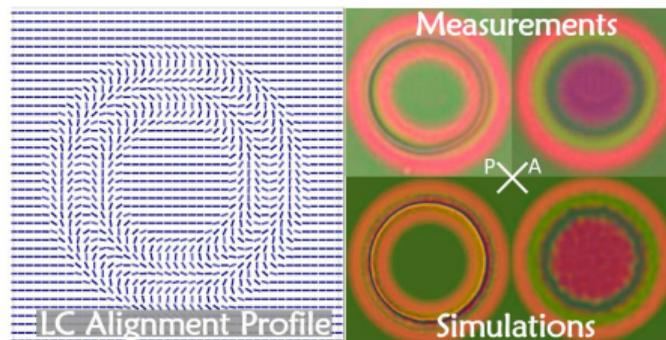


Diffraction op. $D \sim \Delta_\perp$

Advantage: fast and accurate simulations

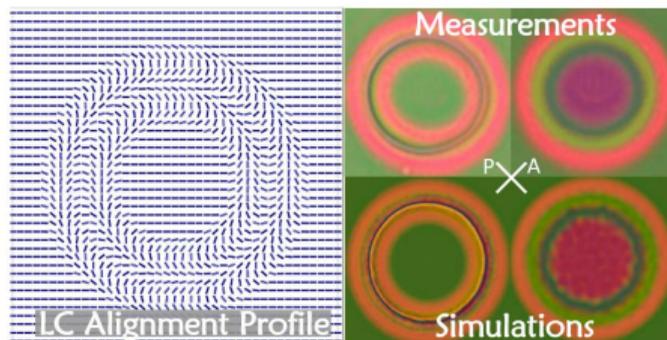
Nemaktis: an open-source package for polarised microscopy

- The open-source package includes:
 - Low-level simulation backends (C++, python)
 - An easy-to-use high-level interface (python)
 - A graphical interface for micrographs simulation



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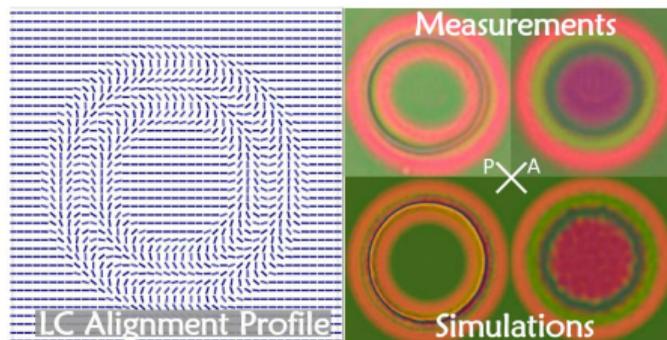
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- Where to find it: search **Nemaktis** on github.com or [google](https://www.google.com).

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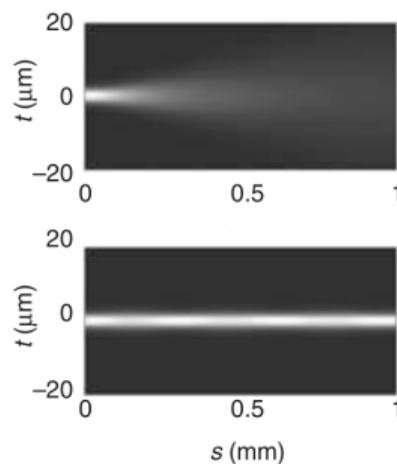
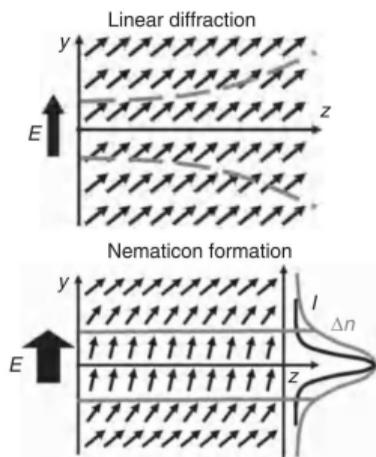
- Where to find it: search **Nemaktis** on github.com or [google](https://www.google.com).
- Closed-source BPM code for advanced uses: wide-angle beam deflection, non-linear optics, etc.

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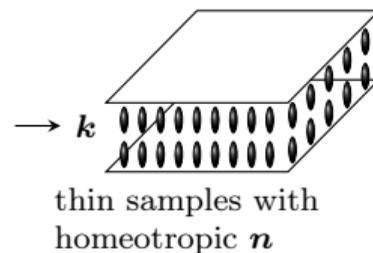
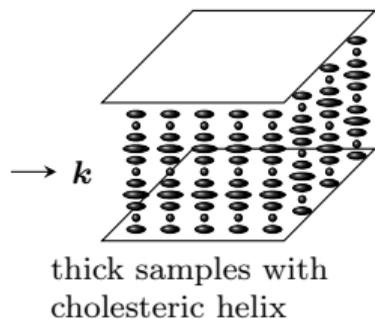
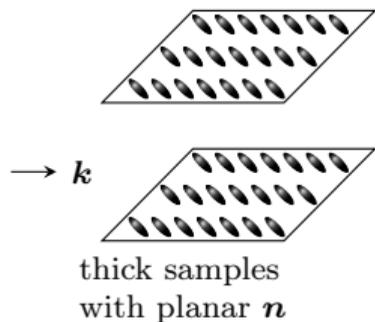
Spatial light solitons in liquid crystals: nematics



G. Assanto. *Nematicons*. John Wiley & Sons, 2013

Motivations

Studied systems in the past 20 years:



What about confined chiral systems? Can we amplify the optical response with chirality?

Orientational elasticity and non-linear interactions

Free energy of the liquid crystal phase:

$$F[\mathbf{n}, \mathbf{E}] = \int_V dV \left[f_F(\mathbf{n}, \nabla \mathbf{n}) - \frac{\epsilon_0 \epsilon_a |\mathbf{n} \cdot \mathbf{E}|^2}{4} \right]$$

Orientational elasticity and non-linear interactions

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Non-linear iterative scheme:

- \mathbf{E}_{k+1} : BPM solution with $\boldsymbol{\epsilon} = \epsilon_{\perp} \mathbf{I} + \epsilon_a \mathbf{n}_k \otimes \mathbf{n}_k$
- $\mathbf{n}_{k+1} = \mathbf{n}_k + \mu \frac{\delta F}{\delta \mathbf{n}} [\mathbf{n}_k, \mathbf{E}_{k+1}]$

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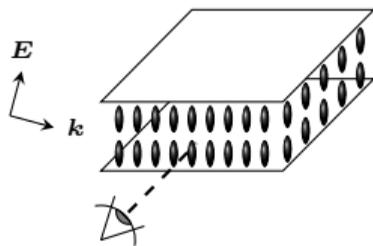
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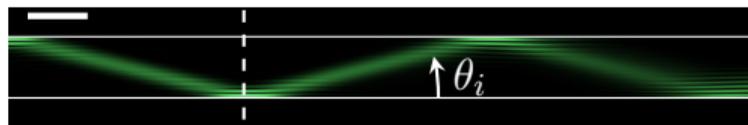
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Typical running time for a mesh of 3×10^6 points: **4 s / step**
 (Full resolution of Maxwell equations for the same mesh: ~ 1 h)

Side-view observations



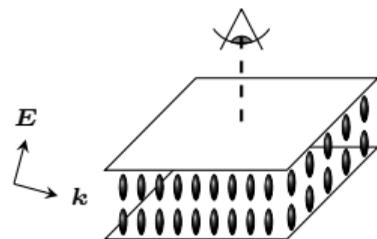
Side slice of beam intensity (simulation):



Side slice of 3PF signal (experiment):



Top-view observations



Top view of the thickness-averaged laser intensity (simulation):

Linear optical regime



Non-linear optical regime

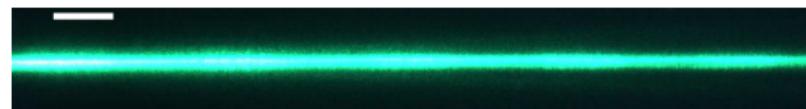


Top view of the scattered laser light (experiments):

Linear optical regime

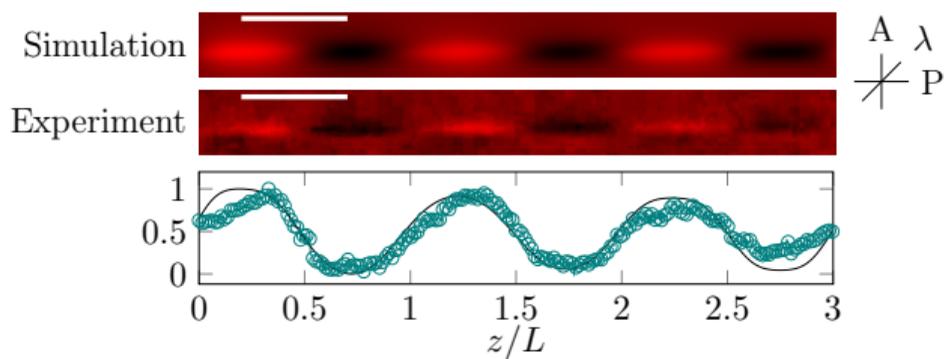


Non-linear optical regime

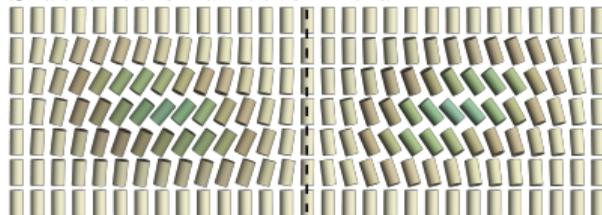


Top-view observations

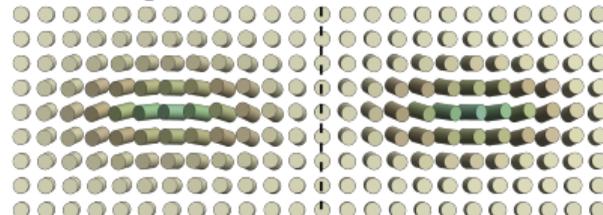
Top view polarised optical micrograph:



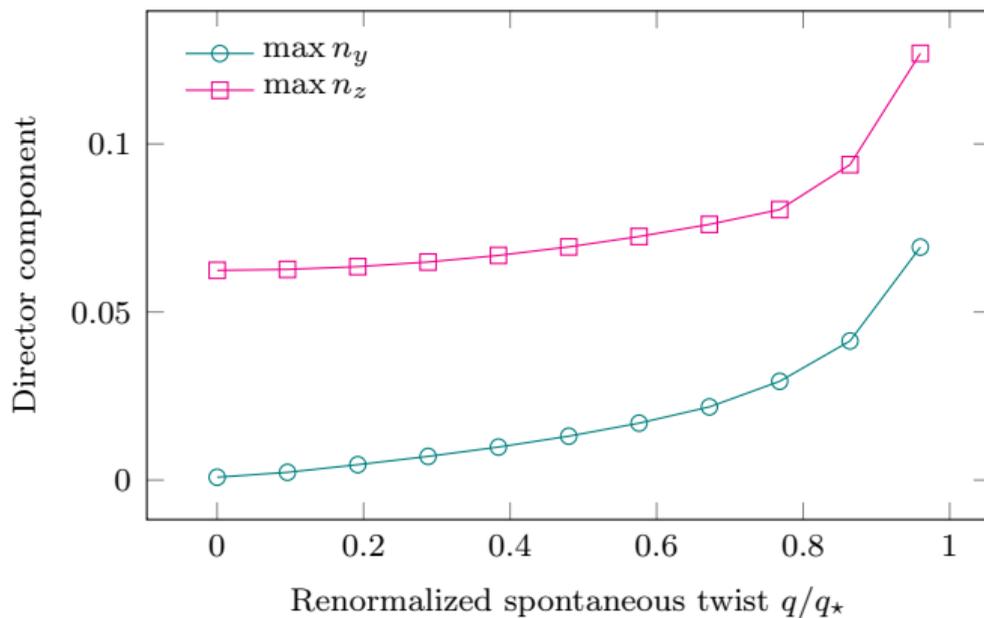
Side slice of director field



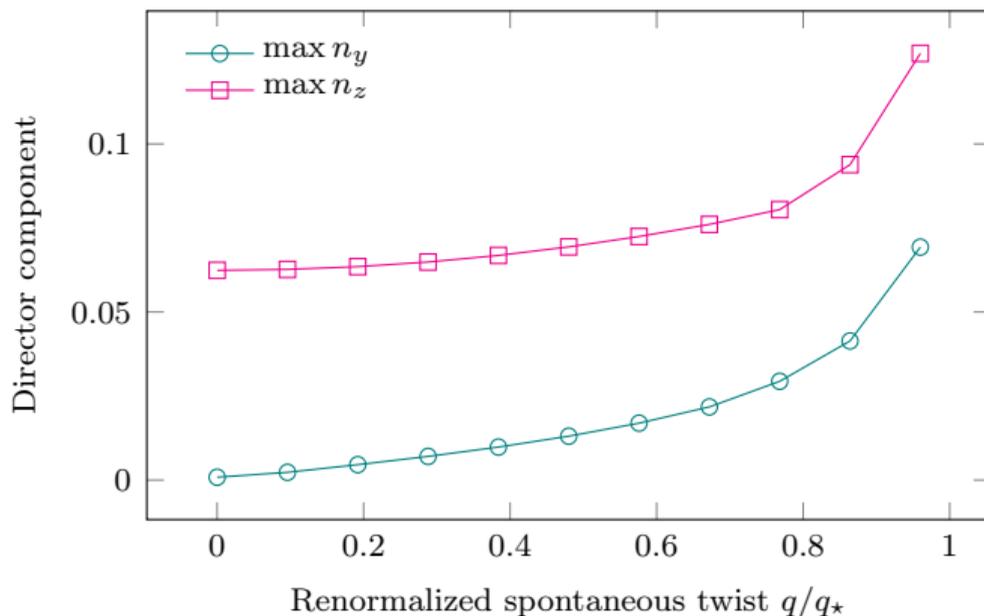
Mid-sample slice of director field



Chirality-enhanced non-linear optical response



Chirality-enhanced non-linear optical response

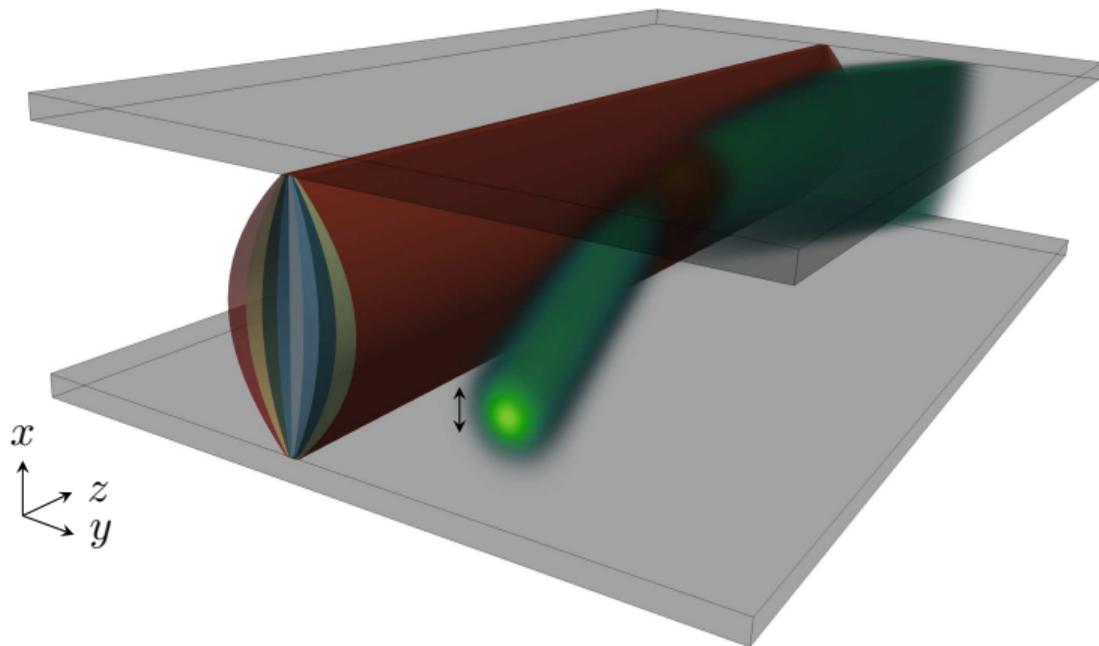


⇒ Potential for low-power non-linear optical photonics devices (e.g. active lenses)

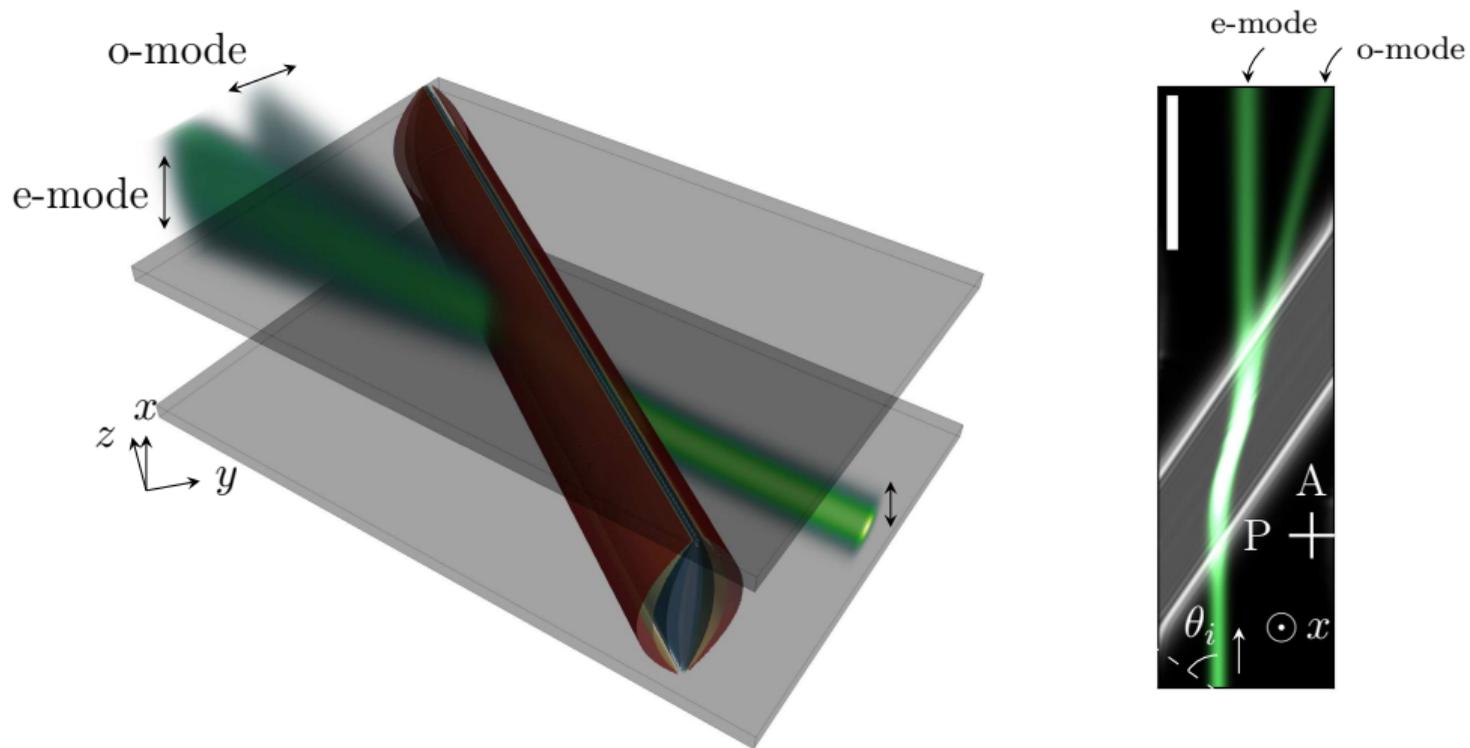
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Transmission and/or reflection with line-like structure

Reflection of incident extraordinary beam ($\theta_i = 70^\circ$):

Transmission and/or reflection with line-like structure

Transmission of incident extraordinary beam ($\theta_i = 55^\circ$):

Description with a generalization of Snell's law

From an exact eigenmode decomposition of Maxwell equations:

$$n^{(\alpha,m)} \sin \theta^{(\alpha,m)} = n_i \sin \theta_i$$

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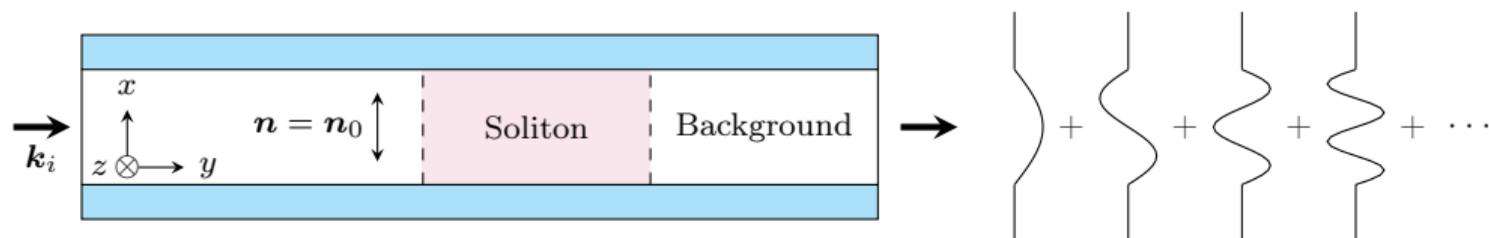
- Usual Snell law: n is the refractive index of an isotropic medium

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- In our system, $n^{(\alpha,m)}$ = effective index of eigenmode $\{\alpha, m\}$ far from the soliton
 - ★ $\alpha = e, o$: polarisation state
 - ★ $m = 1, 2, \dots$: mode index

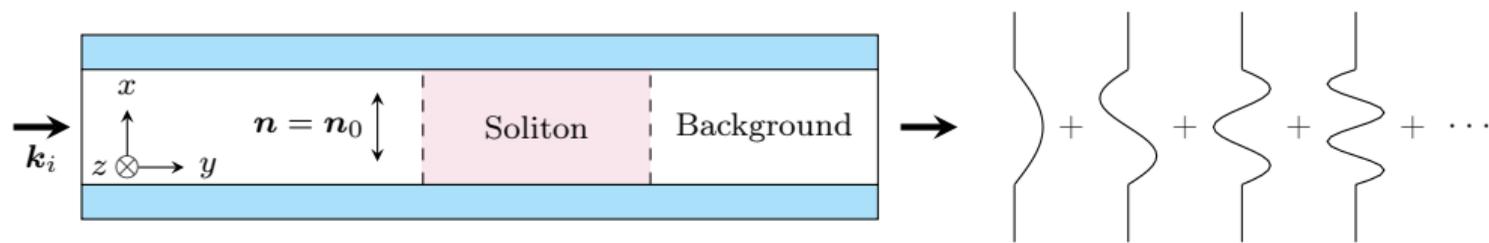


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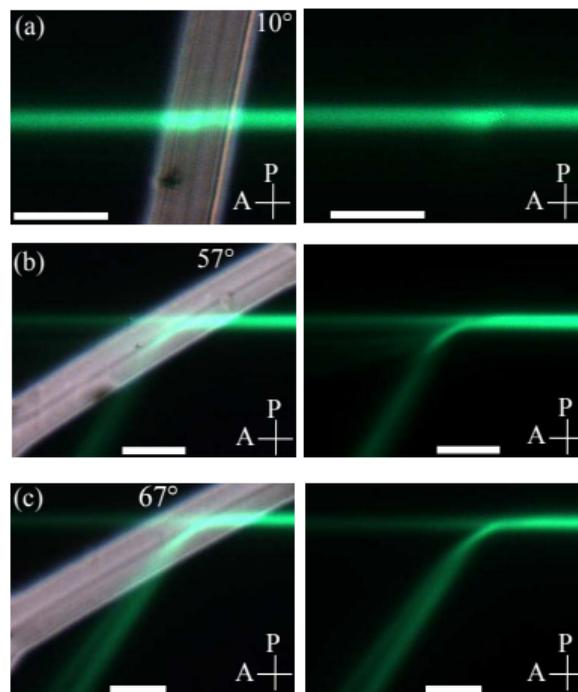
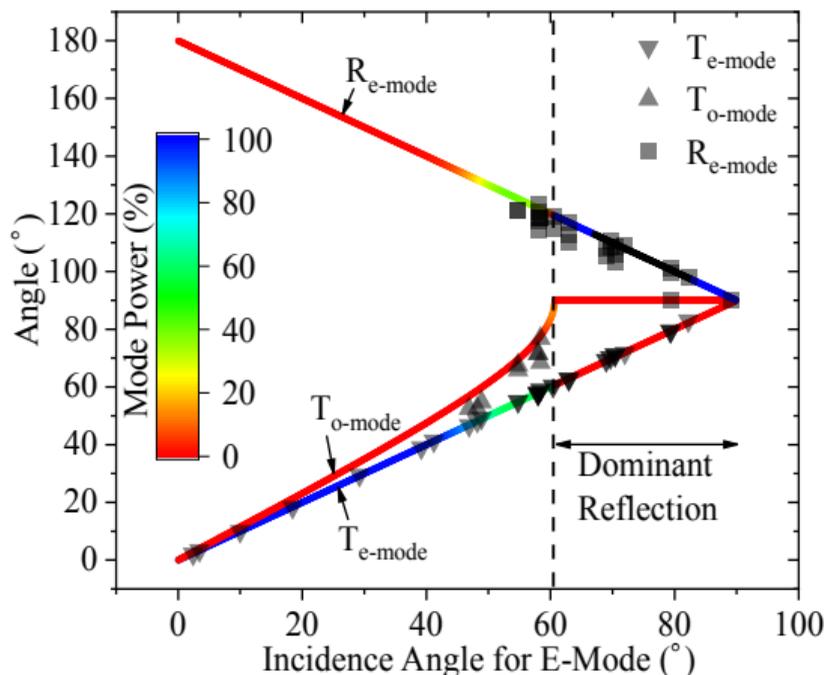
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$\theta^{(\alpha,m)}$ does not depend on the choice of topological soliton!
(but Fresnel coefficients do)

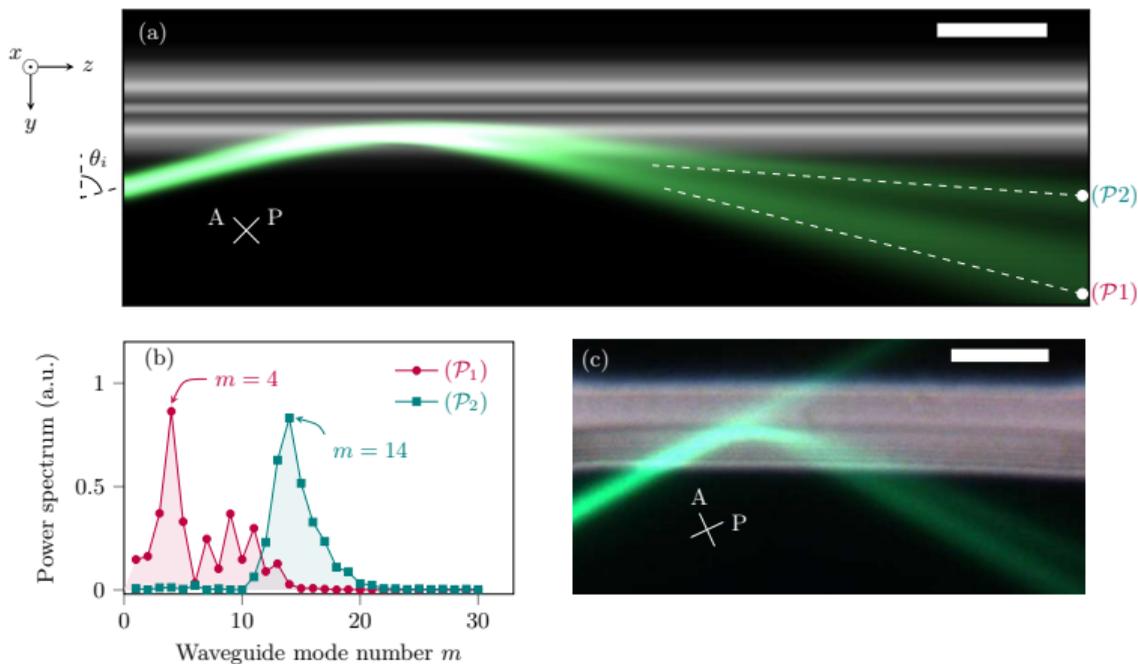
Comparison with experiments

Small mode index approximation in thick samples: $n^{(\alpha,m)} \approx n_\alpha \sqrt{1 - (m/m_0)^2} \approx n_\alpha$



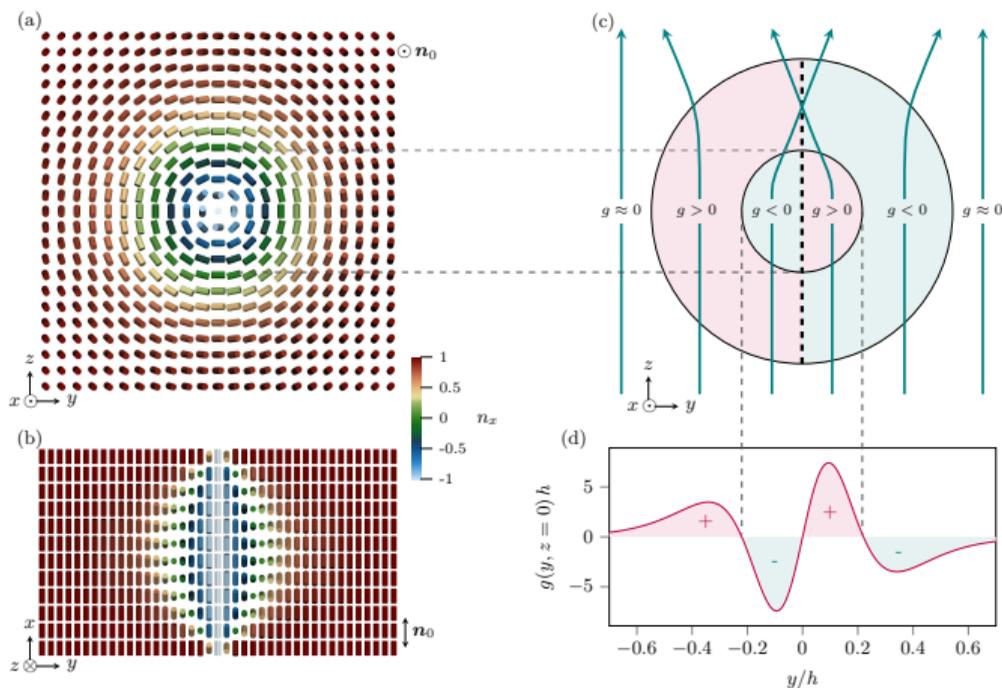
Comparison with experiments

Splitting of eigenmode packets (strongly depends on x-profile): $n^{(\alpha, m_1)} \neq n^{(\alpha, m_2)}$

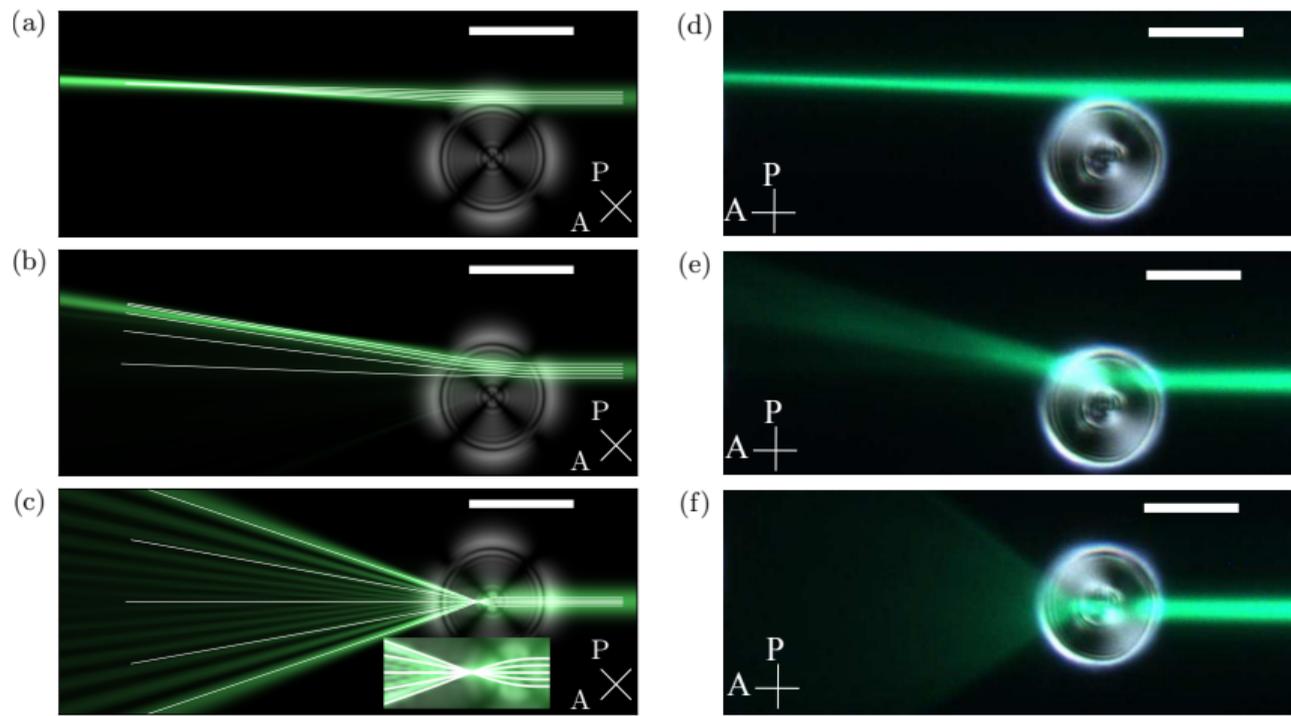


Interaction with point-like topological solitons

Simplification with 2D rays: $dp_y/dz \approx -(\epsilon_a/2n_0)g$, where $g \equiv \partial n_z^2/\partial y$



Light deflection and lensing with pinned torons

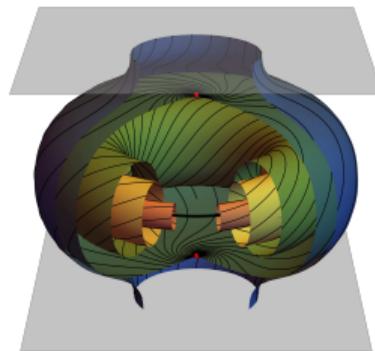
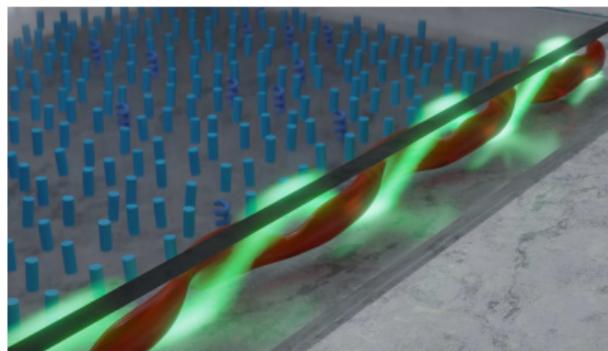


A. Hess *et al.*, *Physical Review X* **10** (2020)

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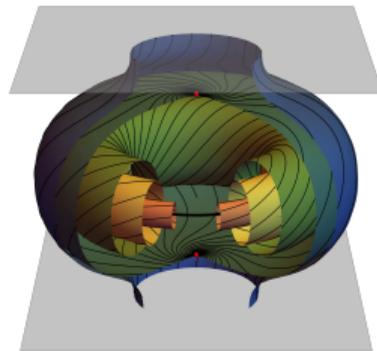
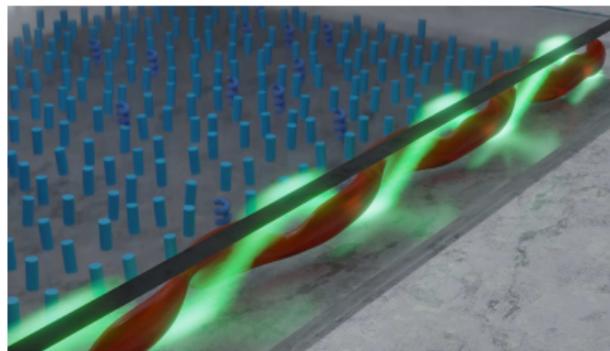
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Take-home message



Chirality and soft topological solitons unlocks new possibilities to control the flow of light at the microscopic level

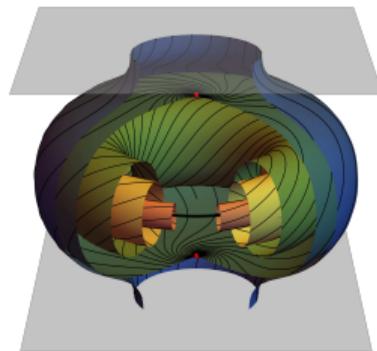
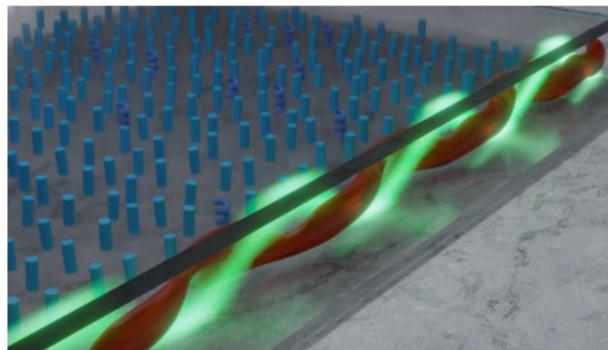
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- Chirality-enhanced optical response: towards enriched opto-mechanical interactions

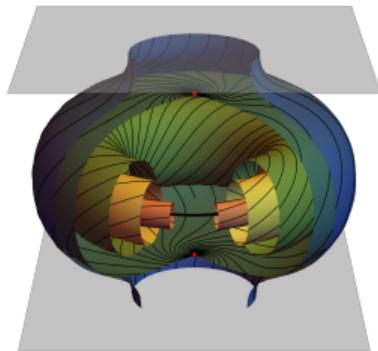
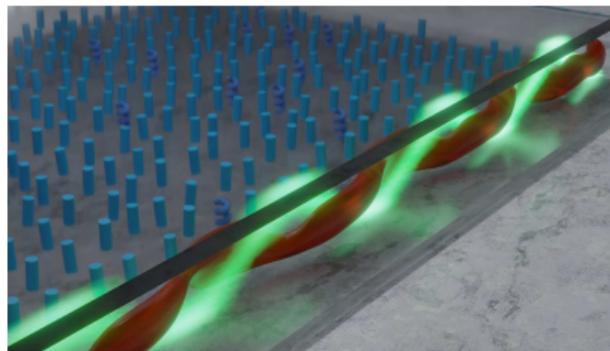
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- Chirality-enhanced optical response: towards enriched opto-mechanical interactions
- Matter transforming light, light transforming matter: what happens when we combine everything?

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- Chirality-enhanced optical response: towards enriched opto-mechanical interactions
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Next step: establishment of a general chirality-enhanced topological optomechanics framework.