Control of the flow of light with soft topological solitons

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Outline

1 Introduction

2 Interaction with line-like solitons

3 Interaction with point-like solitons

4 Summary
The cholesteric phase

- Nematic liquid crystal: no positional order, mean molecular orientation $\mathbf{n}$
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$T_{sol}$ $T_{liq}$ $T$

cholesteric phase  coexistence  isotropic phase
Two classes of soft topological solitons in confined homeotropic samples

- Thin cholesteric layer: unwound background state $n = n_0$
Two classes of soft topological solitons in confined homeotropic samples

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- At intermediate sample thickness:

(a) $n_0$
(b) $n_0$
(c) $n_0$
(d) $n_0$
Two classes of soft topological solitons in confined homeotropic samples

Localized robust birefringent structures $\Rightarrow$ interesting interaction with light?
Light propagation tools in birefringent media

Experimental work in the group of Prof. Smalyukh, modeling work in Ljubljana.

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- Custom-built beam propagation code:
  - Wave-equation in anisotropic media: \[ \partial_k \partial_k \delta_{ij} - \partial_i \partial_j + k_0^2 \epsilon_{ij} \] \[ E_j = 0 \]

Applications:
- Open-source code for polarized optical micrograph simulation (google search: Nemaktis)
- Closed-source code for wide-angle simulations (or non-linear optics)

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  - After eliminating $E_z$ and keeping only forward modes:
    \[
    i \partial_z E_\perp = -\mathcal{P} E_\perp
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- What’s inside \( \mathcal{P} \)?

Phase op. \( K \sim k_0^2 \epsilon \)
Walkoff op. \( W \sim (\epsilon u_z) \otimes \nabla_\perp \)
Diffraction op. \( D \sim \Delta_\perp \)
Introduction

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Transmission and/or reflection

Reflection of incident extraordinary beam ($\theta_i = 70^\circ$):
Transmission and/or reflection

Transmission of incident extraordinary beam ($\theta_i = 55^\circ$):
Description with a generalization of Snell’s law

From an exact eigenmode decomposition of Maxwell equations:

\[ n^{(\alpha,m)} \sin \theta^{(\alpha,m)} = n_i \sin \theta_i \]
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\( \theta^{(\alpha,m)} \) does not depend on the choice of topological soliton! (but Fresnel coefficients do)
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- In our system, \( n(\alpha,m) \) = effective index of eigenmode \( \{\alpha,m\} \) far from the soliton
  - \( \alpha = e, o \): polarisation state
  - \( m = 1, 2, \ldots \): mode index

\[ \begin{array}{c}
\text{Soliton} \\
\text{Background}
\end{array} \]

\[ k_i \]

\[ n = n_0 \]

\[ x \]

\[ y \]

\[ z \]

\[ + + + + \cdots \]
Interaction with line-like solitons

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Comparison with experiments

Small mode index approximation in thick samples: \( n^{(\alpha,m)} \approx n_\alpha \sqrt{1 - (m/m_0)^2} \approx n_\alpha \)
Comparison with experiments

Splitting of eigenmode packets (strongly depends on x-profile): $n^{(\alpha,m_1)} \neq n^{(\alpha,m_2)}$
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Ray-tracing description

Hamiltonian reformulation of century-old Fermat-Grandjean theory:

\[
\frac{dr}{ds} = \frac{\partial H^{(\alpha)}}{\partial p} \quad \frac{dp}{ds} = -\frac{\partial H^{(\alpha)}}{\partial r}
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Ray-tracing description

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- Canonical variables \( \{r, p\} \): position and momentum of "light bullets".

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- Canonical variables \( \{r, p\} \): position and momentum of "light bullets".
- Hamiltonian for ordinary and extraordinary rays:

\[
\begin{align*}
\mathcal{H}^{(o)} &= \frac{|p|^2}{2\epsilon_\perp} \\
\mathcal{H}^{(e)} &= \frac{\epsilon_\perp |p|^2 + \epsilon_a |n(r) \cdot p|^2}{2\epsilon_\perp \epsilon_\parallel}
\end{align*}
\]

Ray-tracing description

Simplification with 2D rays: $\frac{dp_y}{dz} \approx - \left( \epsilon_a / 2n_0 \right) g$, where $g \equiv \partial n_x^2 / \partial y$
Interaction with point-like solitons

Light deflection and lensing with pinned torons
Extension to the nonlinear optical regime with mobile torons

\[ \Delta p \equiv p_f - p_i \]

\[ \delta n^{(TS)} \neq 0 \]

\[ |\delta n^{(TS)}| \geq 0.05 \]

\[ |\delta n^{(EM)}| > 0 \]
Interaction with point-like solitons

Toron trajectories around ‘bouncing’ optical soliton
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Take-home message

Soft topological solitons can reliably control the flow of light at the microscopic level

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- Modeling techniques: Snell law, ray-tracing, beam propagation.

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- Modeling techniques: Snell law, ray-tracing, beam propagation.
- Applications: logical gates, guided self-assembly
Realization and Application of Topological Defect Patterns in Soft and Living Matter

Guest Editors
Dr. Simon Ćopor, Dr. Guilhem Poy, Prof. Dr. Anupam Sengupta

Deadline
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mdpi.com/si/60078
Thank you for your attention!
Total internal reflection can be bypassed by "sliping" under the CF’s defects:

![Diagram showing total internal reflection bypass](image)