Light simulation approaches in birefringent materials

Guilhem Poy

Faculty of Physics and Mathematics, Ljubljana

Septembre 11, 2021

Univerza *v Ljubljani*
Fakulteta za *matematiko in fiziko*

arrs
JAVNA AGENCIJA ZA RAZISKOVALNO DEJAVNOST REPUBLIKE SLOVENIJE

MARIE CURIE ACTIONS

Motivations

- Recent advances in LC-based light application: tunable microresonators, micro-optical elements, diffraction gratings...

Motivations

- Recent advances in LC-based light application: tunable microresonators, micro-optical elements, diffraction gratings...
- Simulation tools for light propagation:
  - ⋆ Jones method (fast but inacurate, easy to code)
  - ⋆ Finite Difference Time Domain (acurate but slow, open-source, complex to use)
  - ⋆ Other methods (in-house implementation)

Motivations

- Recent advances in LC-based light application: tunable microresonators, micro-optical elements, diffraction gratings...
- Simulation tools for light propagation:
  - ⋆ Jones method (fast but inacurate, easy to code)
  - ⋆ Finite Difference Time Domain (acurate but slow, open-source, complex to use)
  - ⋆ Other methods (in-house implementation)

Nemaktis: an easy-to-use open-source platform including tools for light propagation in arbitrary birefringent media.

Outline

Ray-tracing description

Hamiltonian reformulation of century-old Fermat-Grandjean theory:

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\bar{s}} = \frac{\partial \mathcal{H}^{(\alpha)}}{\partial \boldsymbol{p}}$$

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}\bar{s}} = -\frac{\partial \mathcal{H}^{(\alpha)}}{\partial \boldsymbol{r}}$$

G. Poy and S. Žumer, *Soft Matter* **15** (2019)

Ray-tracing description

Hamiltonian reformulation of century-old Fermat-Grandjean theory:

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\bar{s}} = \frac{\partial \mathcal{H}^{(\alpha)}}{\partial \boldsymbol{p}}$$
$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}\bar{s}} = -\frac{\partial \mathcal{H}^{(\alpha)}}{\partial \boldsymbol{r}}$$

- Canonical variables $\{\boldsymbol{r}, \boldsymbol{p}\}$: position and momentum of "light bullets".

G. Poy and S. Žumer, *Soft Matter* **15** (2019)

Ray-tracing description

Hamiltonian reformulation of century-old Fermat-Grandjean theory:

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\bar{s}} = \frac{\partial \mathcal{H}^{(\alpha)}}{\partial \boldsymbol{p}}$$

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}\bar{s}} = -\frac{\partial \mathcal{H}^{(\alpha)}}{\partial \boldsymbol{r}}$$

- Canonical variables $\{\boldsymbol{r}, \boldsymbol{p}\}$: position and momentum of "light bullets".
- Hamiltonian for ordinary and extraordinary rays:

$$\mathcal{H}^{(o)} = \frac{|\boldsymbol{p}|^2}{2\epsilon_\perp}$$

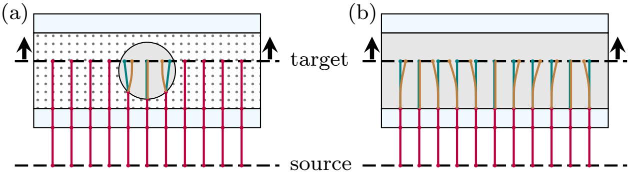
$$\mathcal{H}^{(e)} = \frac{\epsilon_\perp |\boldsymbol{p}|^2 + \epsilon_a \left|\boldsymbol{n}(\boldsymbol{r}) \cdot \boldsymbol{p}\right|^2}{2\epsilon_\perp \epsilon_\|}$$

G. Poy and S. Žumer, *Soft Matter* **15** (2019)

Energy transport and conservation law

$$\frac{\mathrm{d}\eta}{\mathrm{d}\bar{s}} = \{\boldsymbol{\eta}, \mathcal{H}\}$$

$$\boldsymbol{\eta} \equiv (\boldsymbol{x}, \boldsymbol{p})$$

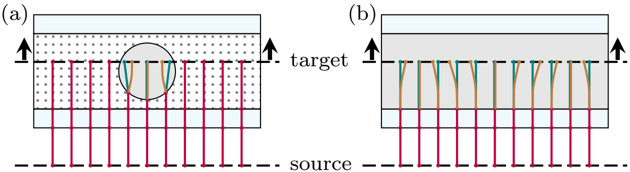


G. Poy and S. Žumer, *Soft Matter* **15** (2019)

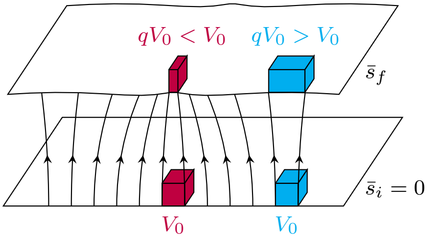
Energy transport and conservation law

$$\frac{d\eta}{d\bar{s}} = \{\boldsymbol{\eta}, \mathcal{H}\}$$

$$\boldsymbol{\eta} \equiv (\boldsymbol{x}, \boldsymbol{p})$$

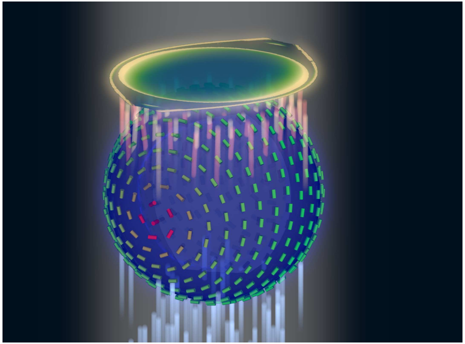


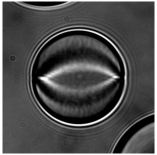
$\mathcal{F}^{(\alpha)} = n_{\text{eff}} \sqrt{q}\, E$ conserved along a ray



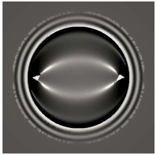
G. Poy and S. Žumer, *Soft Matter* **15** (2019)

Application to bright-field microscopy



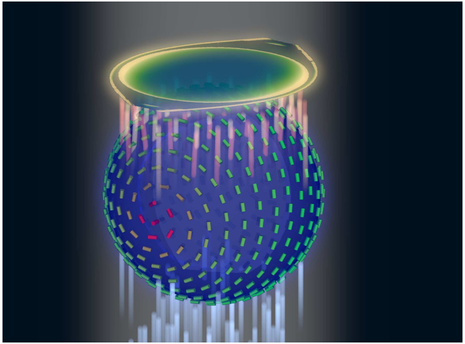


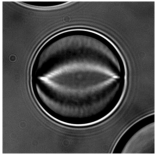
exp.



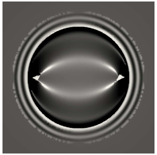
sim.

Application to bright-field microscopy





exp.

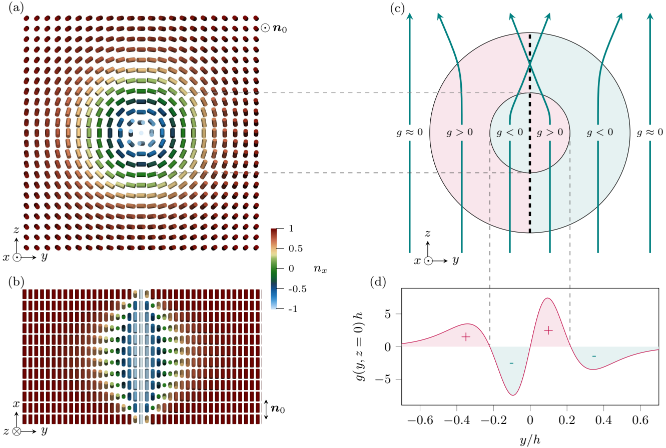


sim.

Advantage: access to ray geometry and natural eigenmodes
Disadvantage: Mauguin regime, caustics

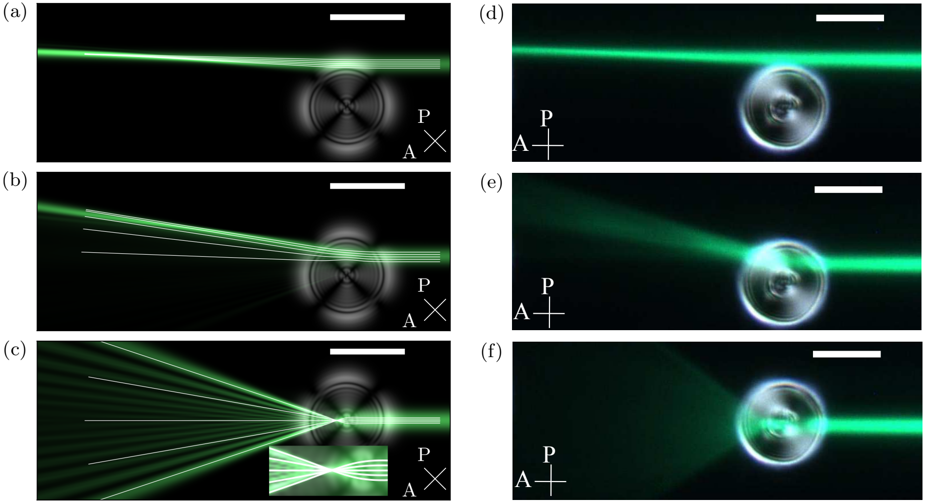
Application to light-matter interactions with torons

Simplification with 2D rays: $\mathrm{d}p_y/\mathrm{d}z \approx -\left(\epsilon_a/2n_0\right) g$, where $g \equiv \partial n_z^2/\partial y$



A. Hess *et al.*, *Physical Review X* **10** (2020)

Application to light-matter interactions with torons



Outline

Physics-based splitting of the wave equation

- Wave-equation in anisotropic media: $\left[\partial_k \partial_k \delta_{ij} - \partial_i \partial_j + k_0^2 \epsilon_{ij}\right] E_j = 0$

G. Poy and S. Žumer, *Optics Express* **28** (2020)

Physics-based splitting of the wave equation

- Wave-equation in anisotropic media: $\left[\partial_k\partial_k\delta_{ij} - \partial_i\partial_j + k_0^2\epsilon_{ij}\right] E_j = 0$
- After eliminating E_z and keeping only forward modes:

$$i\partial_z \boldsymbol{E}_\perp = -\boldsymbol{\mathcal{P}}\boldsymbol{E}_\perp$$

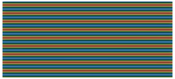
G. Poy and S. Žumer, *Optics Express* **28** (2020)

Physics-based splitting of the wave equation

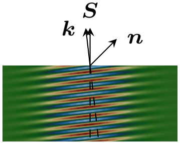
- Wave-equation in anisotropic media: $\left[\partial_k \partial_k \delta_{ij} - \partial_i \partial_j + k_0^2 \epsilon_{ij}\right] E_j = 0$
- After eliminating E_z and keeping only forward modes:

$$i\partial_z \boldsymbol{E}_\perp = -\boldsymbol{\mathcal{P}} \boldsymbol{E}_\perp$$

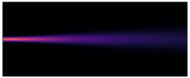
- What's inside $\boldsymbol{\mathcal{P}}$?



Phase op. $\boldsymbol{K} \sim k_0^2 \boldsymbol{\epsilon}$



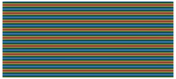
Walkoff op. $\boldsymbol{W} \sim (\boldsymbol{\epsilon}\, \boldsymbol{u}_z) \otimes \boldsymbol{\nabla}_\perp$



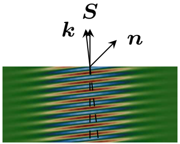
Diffraction op. $\boldsymbol{D} \sim \boldsymbol{\Delta}_\perp$

G. Poy and S. Žumer, *Optics Express* **28** (2020)

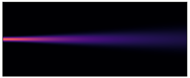
Beam propagation formula



Phase op. $\boldsymbol{K} \sim k_0^2 \boldsymbol{\epsilon}$



Walkoff op. $\boldsymbol{W} \sim (\boldsymbol{\epsilon}\, \boldsymbol{u}_z) \otimes \boldsymbol{\nabla}_\perp$



Diffraction op. $\boldsymbol{D} \sim \boldsymbol{\Delta}_\perp$

General expression for $\boldsymbol{\mathcal{P}}$:

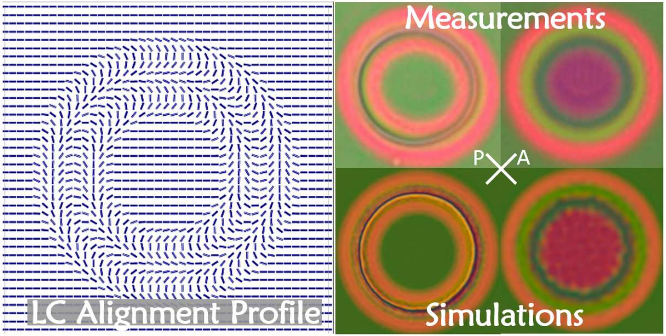
$$\boldsymbol{\mathcal{P}} = i\,\boldsymbol{W} + \sqrt{\boldsymbol{K} + \boldsymbol{D}} + \mathcal{O}\left(\delta\epsilon^2\right)$$

Explicit solution for the transverse optical field:

$$\boldsymbol{E}_\perp\Big|_{z_2} = \exp\left\{ i \int_{z_1}^{z_2} \boldsymbol{\mathcal{P}} \mathrm{d}z \right\} \boldsymbol{E}_\perp\Big|_{z_1}$$

G. Poy and S. Žumer, *Optics Express* **28** (2020)

Application to polarised micrographs simulation



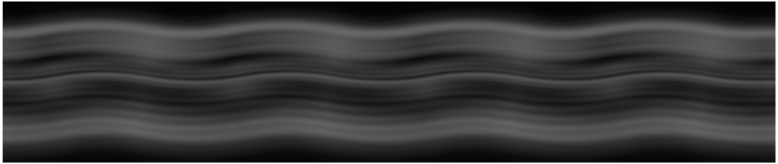
Advantage: fast and accurate simulations

B. Berteloot *et al.*, *Soft Matter* **16** (2020)

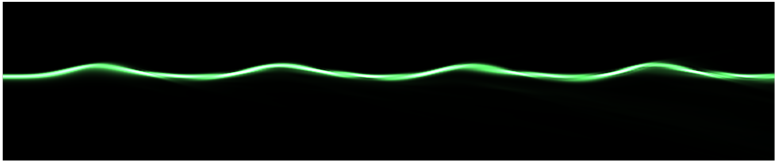
Application to light waveguiding

Simulated light mode inside a curved cholesteric finger of type II:

(a)



(b)



G. Poy and S. Žumer, *Optics Express* **28** (2020)

Outline

Availability as an open-source package: Nemaktis

- The open-source package (Windows/Linux) includes:
  - Low-level simulation backends (C++, python)
  - An easy-to-use high-level interface (python)
  - A graphical interface for micrographs simulation

Availability as an open-source package: Nemaktis

- The open-source package (Windows/Linux) includes:
  - Low-level simulation backends (C++, python)
  - An easy-to-use high-level interface (python)
  - A graphical interface for micrographs simulation
- Where to find it: search **Nemaktis** on **github.com** or **google**.

Availability as an open-source package: Nemaktis

- The open-source package (Windows/Linux) includes:
  - Low-level simulation backends (C++, python)
  - An easy-to-use high-level interface (python)
  - A graphical interface for micrographs simulation
- Where to find it: search **Nemaktis** on **github.com** or **google**.
- Closed-source BPM code for advanced uses: wide-angle beam deflection, non-linear optics, etc.

Thank you for your attention!