A new operator-splitted beam propagation method with application to non-linear optics in liquid crystals

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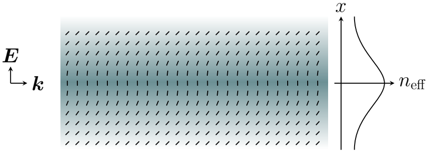
Outline

Motivations

- Spatial light solitons in liquid crystals: nematicons



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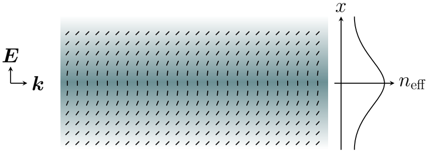

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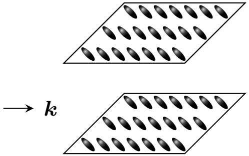
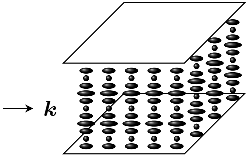
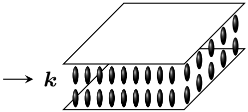


thick samples with planar $\boldsymbol{n}$



thick samples with cholesteric helix



thin samples with homeotropic $\boldsymbol{n}$

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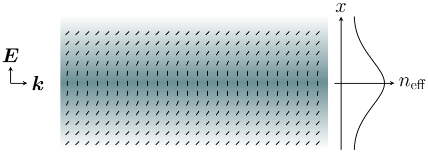

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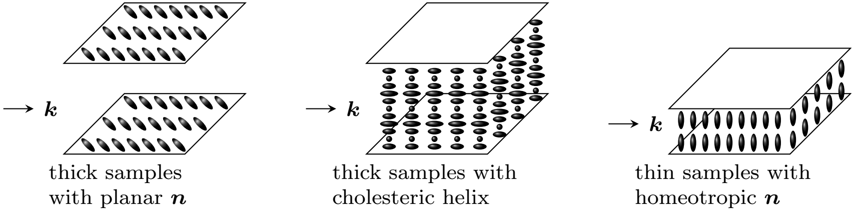
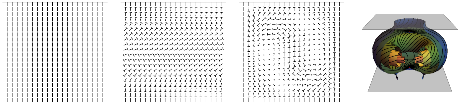


What about confined chiral systems?

Motivations

What makes frustrated cholesteric (FCLC) an interesting system:

- Metastability for carefully chosen values of d/P



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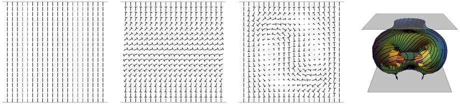
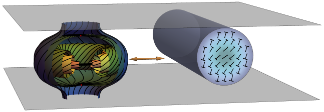


- Rich possibilities of interaction between light beams and topological solitons.



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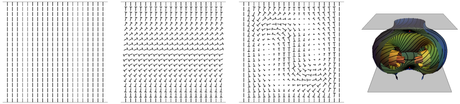

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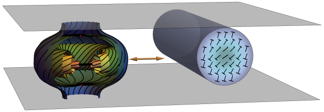
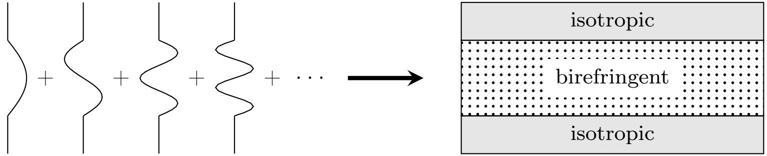


Problematics

- How to accurately model light propagation in confined birefringent systems?
- Can we generate light solitons in frustrated cholesteric?

Outline

Required features



$$\boldsymbol{E}(\boldsymbol{r}_\perp, z + dz) = \mathbf{U}(z, dz)\, \boldsymbol{E}(\boldsymbol{r}_\perp, z)$$

- Diffraction at interfaces of discontinuity of the permittivity tensor

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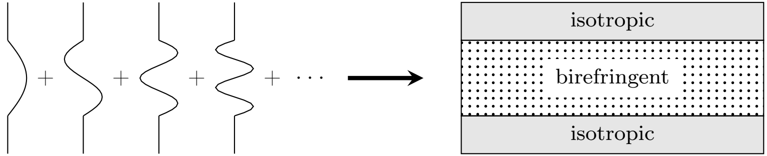

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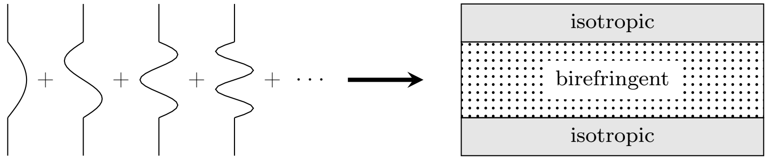

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- Diffraction at interfaces of discontinuity of the permittivity tensor
- Wide-angle propagation
- Anisotropic effects in LC: beam walk-off, couplings between orthogonal polarisations...

Physics-based operator splitting in the wave equation

From Maxwell equations, after eliminating E_z:

$$\left(i\partial_z + i\mathbf{W} + \mathbf{D}' + \mathbf{D}'' + \sqrt{\mathbf{K}}\right)\boldsymbol{E}_\perp = 0$$

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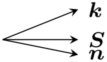

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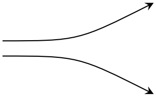

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- $\sqrt{\mathbf{K}}$: Phase operator, exact expression. $\sim$ Jones matrix.

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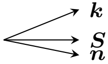

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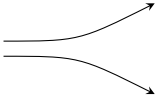

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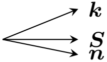

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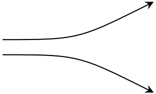

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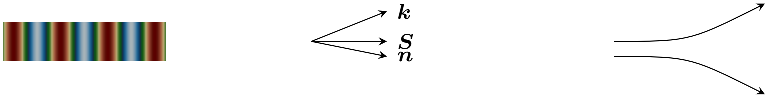

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- $\mathbf{D}''$: Wide-angle operator, fourth-order transverse derivatives.

Orientational elasticity and non-linear interactions

Free energy of the liquid crystal phase:

$$F[\boldsymbol{n},\boldsymbol{E}]=\int_V \mathrm{d}V\left[f_{\mathrm{F}}(\boldsymbol{n},\nabla\boldsymbol{n})-\frac{\epsilon_0\epsilon_a\left|\boldsymbol{n}\cdot\boldsymbol{E}\right|^2}{4}\right]$$

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Non-linear iterative scheme:

- $\boldsymbol{E}_{k+1}$: BPM solution with $\boldsymbol{\epsilon} = \epsilon_\perp \mathbf{I} + \epsilon_a \boldsymbol{n}_k \otimes \boldsymbol{n}_k$
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Typical running time for a mesh of 3×10^6 points: **4 s / step**

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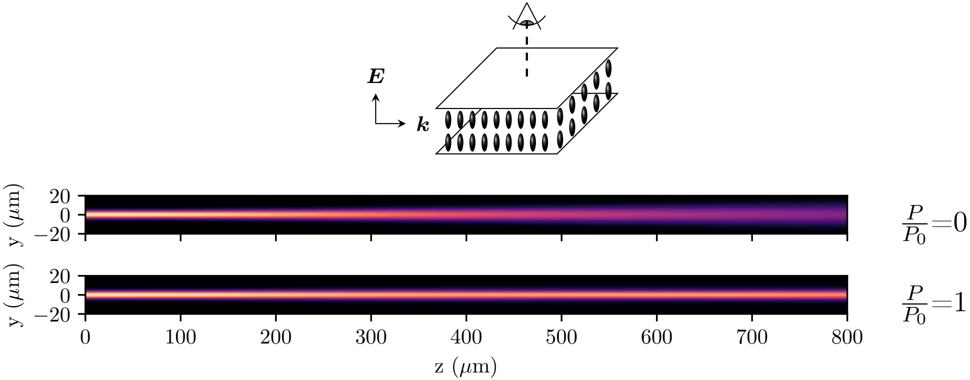
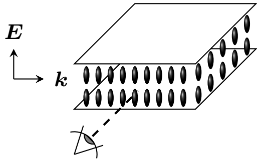
Optical fields structure

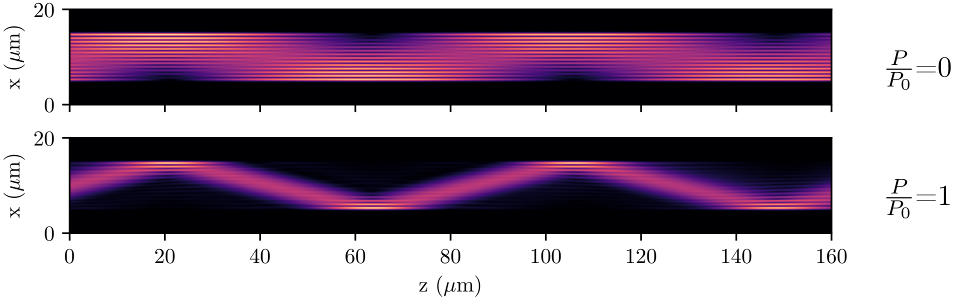
Top view of the thickness-averaged intensity:



Optical fields structure

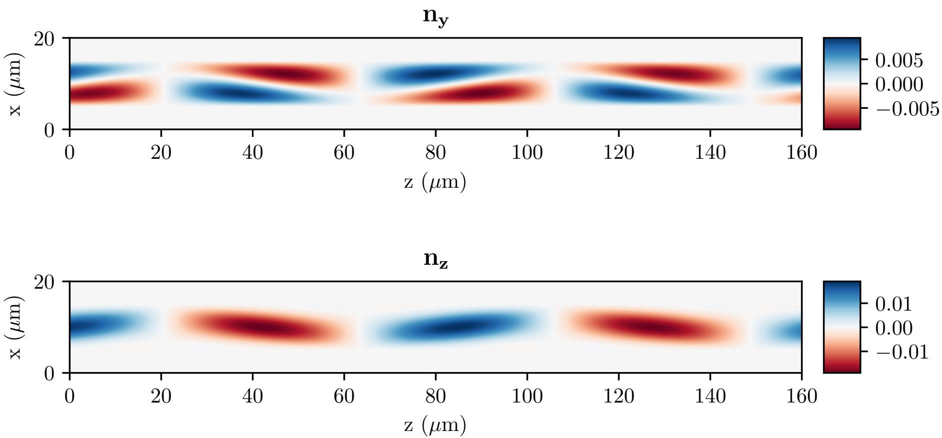
Side view of the field amplitude:





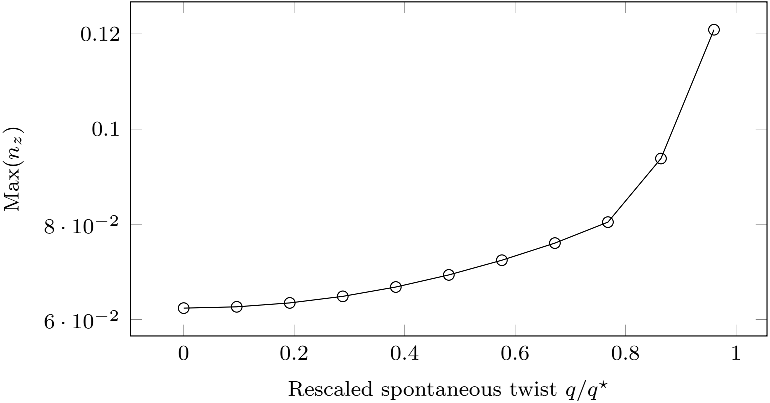
Non-linear response

Side view of the director field:



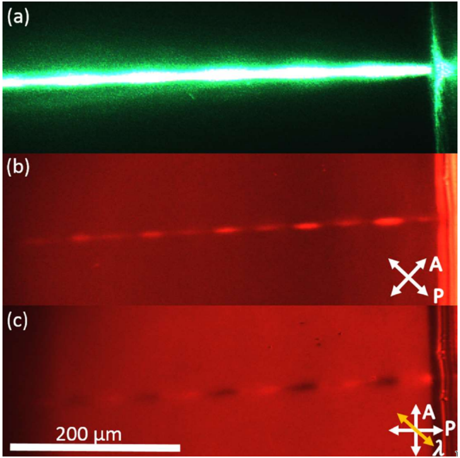
Non-linear response

Chirality-enhanced Kerr response:



Comparison with experiments

Scattered light and polarised optical micrographs (I. Smalyukh group):



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- New BPM method tailored for propagation in confined anisotropic waveguide.
- It is possible to generate solitons in confined cholesteric system, with:
  - ⋆ "bouncing" beam between the sample plates
  - ⋆ periodic reorientation along the beam axis
  - ⋆ chirality-enhanced Kerr response
- To be explored:
  - ⋆ Superposition of normal and transverse polarisations
  - ⋆ Interaction with topological solitons

Thank you for your attention!