A new operator-splitted beam propagation method with application to non-linear optics in liquid crystals

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Outline

1 Motivations

2 Light propagation in confined birefringent media

3 Light solitons in frustrated cholesteric

4 Conclusion
Motivations

- Spatial light solitons in liquid crystals: nematicons

Studied systems in the past 20 years:
- thick samples with planar nematic
- thick samples with cholesteric helix
- thin samples with homeotropic nematic

What about confined chiral systems?

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OLC 2019
Québec
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Motivations

What makes frustrated cholesteric (FCLC) an interesting system:

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- Rich possibilities of interaction between light beams and topological solitons.
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Problematics

- How to accurately model light propagation in confined birefringent systems?
- Can we generate light solitons in frustrated cholesteric?
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Required features

- Diffraction at interfaces of discontinuity of the permittivity tensor
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- Wide-angle propagation
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- Wide-angle propagation
- Anisotropic effects in LC: beam walk-off, couplings between orthogonal polarisations...
Physics-based operator splitting in the wave equation

From Maxwell equations, after eliminating $E_z$:

$$\left( i\partial_z + iW + D' + D'' + \sqrt{K} \right) E_\perp = 0$$
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- $\sqrt{K}$: Phase operator, exact expression. $\sim$ Jones matrix.
- $W$: Walk-off operator, first-order transverse derivative. $\sim$ Translation operator.
- $D'$: Diffraction operator, second-order transverse derivatives. $\sim$ Anisotropic diffusion.
- $D''$: Wide-angle operator, fourth-order transverse derivatives.
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Orientational elasticity and non-linear interactions

Free energy of the liquid crystal phase:

\[ F[n, E] = \int_V dV \left[ f_F(n, \nabla n) - \frac{\epsilon_0 \epsilon_a |n \cdot E|^2}{4} \right] \]
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Non-linear iterative scheme:

- \( E_{k+1} \): BPM solution with \( \varepsilon = \varepsilon_\perp \mathbf{I} + \varepsilon_a n_k \otimes n_k \)
- \( n_{k+1} = n_k + \mu \frac{\delta F}{\delta n} [n_k, E_{k+1}] \)
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Typical running time for a mesh of \( 3 \times 10^6 \) points: \( 4 \text{ s / step} \)
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Optical fields structure

Top view of the thickness-averaged intensity:

\[ \frac{P}{P_0} = 0 \]
\[ \frac{P}{P_0} = 1 \]
Optical fields structure

Side view of the field amplitude:
Non-linear response

Side view of the director field:

\[ n_y \]

\[ n_z \]
Non-linear response

Chirality-enhanced Kerr response:

![Graph of rescaled spontaneous twist against q/q*](image)
Comparison with experiments

Scattered light and polarised optical micrographs (I. Smalyukh group):
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- New BPM method tailored for propagation in confined anisotropic waveguide.
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It is possible to generate solitons in confined cholesteric system, with:
- "bouncing" beam between the sample plates
- periodic reorientation along the beam axis
- chirality-enhanced Kerr response
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To be explored:

- Superposition of normal and transverse polarisations
- Interaction with topological solitons
Thank you for your attention!