Simulation of polarized optical micrographs including light-deviation effects in slowly-varying birefringent structures

Guilhem Poy

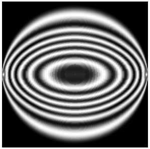
Faculty of Physics and Mathematics, Ljubljana

September 18, 2018

Outline

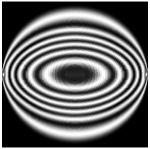
Motivations

Transmission of an arbitrary birefringent sample between polariser and analyzer: Jones method



Motivations

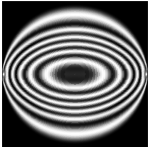
Transmission of an arbitrary birefringent sample between polariser and analyzer: Jones method



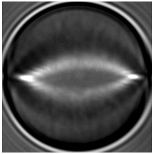
- First limitation: numerical aperture
  $\Rightarrow$ generalized Jones method by Mur et al

Motivations

Transmission of an arbitrary birefringent sample between polariser and analyzer: Jones method



- First limitation: numerical aperture
  ⇒ generalized Jones method by Mur et al
- Second limitation: deflection of the extraordinay rays.
  How to explain the non-zero contrast of natural light micrographs?



Objective

Question

Can we design an efficient method to simulate optical micrographs of LC samples, including light deviation effects?

Objective

Question

Can we design an efficient method to simulate optical micrographs of LC samples, including light deviation effects?

Working hypotheses: $|\nabla \boldsymbol{n}| \sim \frac{1}{L} \ll \frac{1}{\lambda}$ + Mauguin regime

Objective

Question

Can we design an efficient method to simulate optical micrographs of LC samples, including light deviation effects?

Working hypotheses: $|\nabla \boldsymbol{n}| \sim \frac{1}{L} \ll \frac{1}{\lambda}$ + Mauguin regime

Goals of our method:

- Accurate simulation of Maxwell equations: WKB expansion.

Objective

Question

Can we design an efficient method to simulate optical micrographs of LC samples, including light deviation effects?

Working hypotheses: $|\nabla \boldsymbol{n}| \sim \frac{1}{L} \ll \frac{1}{\lambda}$ + Mauguin regime

Goals of our method:

- Accurate simulation of Maxwell equations: WKB expansion.
- Reconstruction of bulk data

Objective

Question

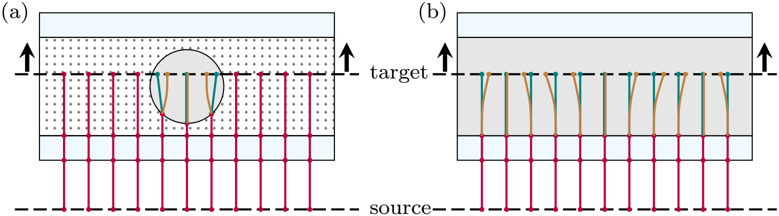
Can we design an efficient method to simulate optical micrographs of LC samples, including light deviation effects?

Working hypotheses: $|\nabla \boldsymbol{n}| \sim \frac{1}{L} \ll \frac{1}{\lambda}$ + Mauguin regime

Goals of our method:

- Accurate simulation of Maxwell equations: WKB expansion.
- Reconstruction of bulk data
- Reconstruction of screen data, focalisation can be adjusted (as in a real microscope)

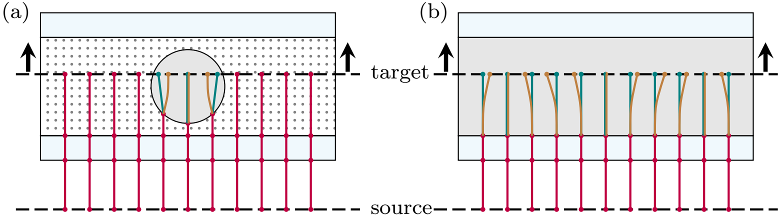
Hamiltonian ray-tracing



- Evolution of isotropic, extraordinary and ordinary rays: Hamilton Eqs.

$$\frac{\mathrm{d}\eta}{\mathrm{d}\bar{s}} = \{\boldsymbol{\eta}, \mathcal{H}\}$$
$$\boldsymbol{\eta} \equiv (\boldsymbol{x}, \boldsymbol{p})$$

Hamiltonian ray-tracing

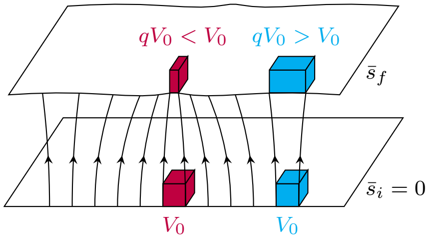


- Evolution of isotropic, extraordinary and ordinary rays: Hamilton Eqs.

$$\frac{\mathrm{d}\eta}{\mathrm{d}\bar{s}} = \{\boldsymbol{\eta}, \mathcal{H}\}$$
$$\boldsymbol{\eta} \equiv (\boldsymbol{x}, \boldsymbol{p})$$

- Discontinuity of the optical index: Fresnel boundary conditions.

Reconstruction of the electric field amplitude

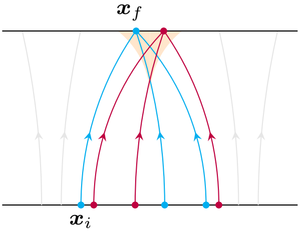


New result:

$\mathcal{F}^{(\alpha)} = n_{\text{eff}}\sqrt{q}\,E$ conserved along a ray of the family $\alpha = e, o, i$.

$\Rightarrow$ Compact statement of the conservation of energy

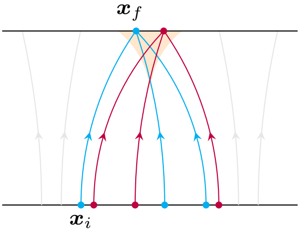
Caustics



Mapping $\boldsymbol{\pi} : \boldsymbol{x}_i \to \boldsymbol{x}_f$:

- No caustics: one-to-one correspondance
- Caustic domains: many-to-one correspondance

Caustics



Mapping $\boldsymbol{\pi} : \boldsymbol{x}_i \to \boldsymbol{x}_f$:

- No caustics: one-to-one correspondance
- Caustic domains: many-to-one correspondance

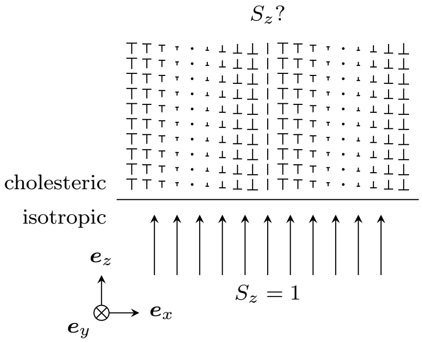
⇒ Necessity of finding **all** source points $\{\boldsymbol{x}_i^{(1)}, \boldsymbol{x}_i^{(2)}, \ldots\}$ for a given target point $\boldsymbol{x}_f$.

(Homotopy continuation algorithm on $\boldsymbol{\pi}(\boldsymbol{x}_i) - \boldsymbol{x}_f$)

Outline

Setup

Incident plane wave on a transverse cholesteric helix:
Poynting vector field $\boldsymbol{S}$ inside the cholesteric phase?

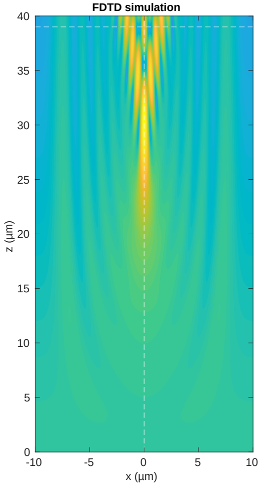


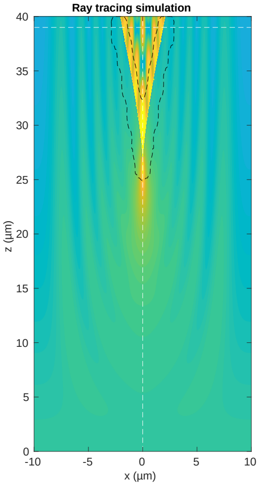
Initial polarisation: $\frac{\boldsymbol{e}_x+\boldsymbol{e}_y}{\sqrt{2}}$

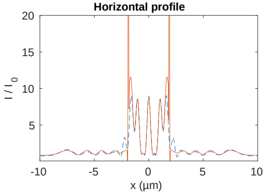
Two methods of resolution:

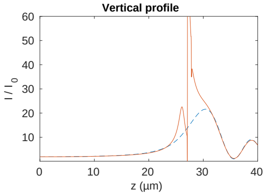
- Our improved ray-tracing method
- Exact resolution of Maxwell Eqs. (FDTD)

Results

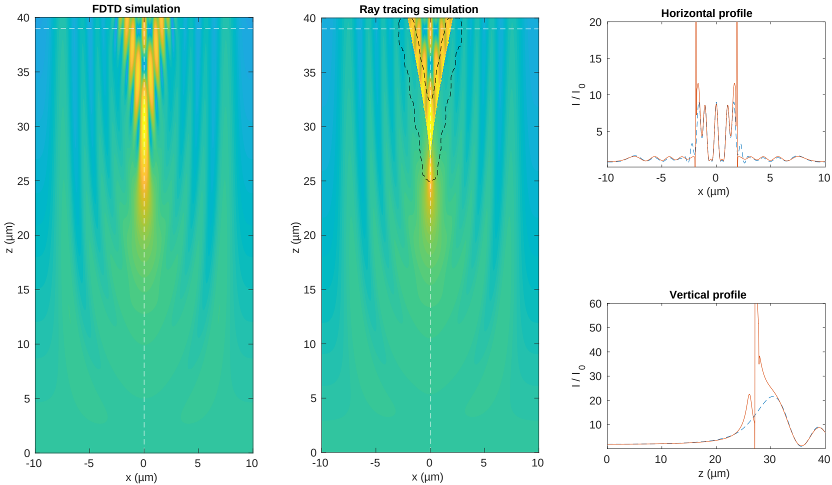

FDTD simulation
z (µm)
x (µm)


Ray tracing simulation
z (µm)
x (µm)


Horizontal profile
I / I0
x (µm)


Vertical profile
I / I0
z (µm)

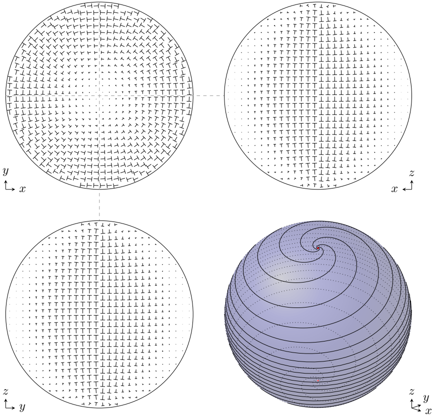
Results



Fast and accurate reconstruction of $\boldsymbol{S}$ far from the caustic boundaries

Outline

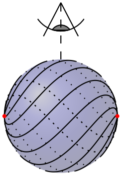
Nematic twisted bipolar droplet (SSY in water)



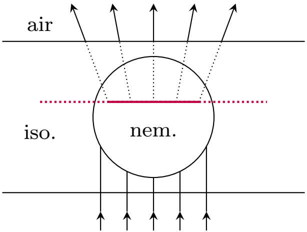
Twisted bipolar droplet in SSY suspensions:

- Planar anchoring: 2 defects
- Double twist (giant elastic anisotropy $K_2 \ll K_{1,3}$)

Orientation in the microscope:



Computation of the natural light micrographs



Projection on a screen through a perfect lens
⇕
Backward propagation to the focalisation plane

Natural light micrographs: average over all polarisation states.

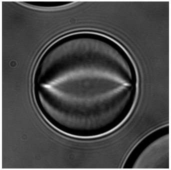
Results

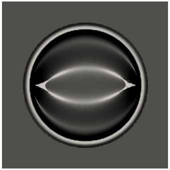
Experiment

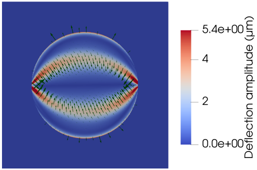
Simulation

Deflection map (extraordinay rays)

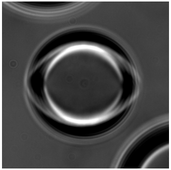
$\Delta z = 0\ \mu\text{m}$

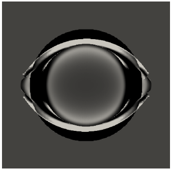


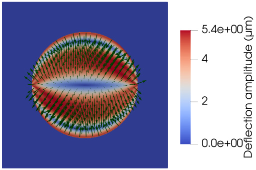




$\Delta z = 50\ \mu\text{m}$







Outline

Conclusion and outlook

- New method with fast and accurate reconstruction of $\boldsymbol{S}$ far from caustics.

Conclusion and outlook

- New method with fast and accurate reconstruction of $\boldsymbol{S}$ far from caustics.
- Good agreement with experimental micrographs of twisted bipolar droplets.

Conclusion and outlook

- New method with fast and accurate reconstruction of $\boldsymbol{S}$ far from caustics.
- Good agreement with experimental micrographs of twisted bipolar droplets.
- Perspectives:
  - ⋆ CIE 1931 color space
  - ⋆ Role of numerical aperture?
  - ⋆ Link between chirality and symmetry-breaking in micrographs?
  - ⋆ New systems: skyrmions, cholesteric fingers, banded droplets...

Going beyond the Mauguin regime

In this talk: $\mathcal{F}^{(e,o)}$ conserved along extraordinary/ordinary rays.

Going beyond the Mauguin regime

In this talk: $\mathcal{F}^{(e,o)}$ conserved along extraordinary/ordinary rays.

Outside the Mauguin regime:

Twist of the (o) polarisation

$$\frac{\mathrm{d}}{\mathrm{d}\bar{s}}\left[\mathcal{F}^{(e)}\right] = -\mathcal{F}^{(o)}\,\mathrm{e}^{-i\Delta\phi}\,\mathcal{T}^{(o)}$$

Twist of the (e) polarisation

$$\frac{\mathrm{d}}{\mathrm{d}\bar{s}}\left[\mathcal{F}^{(o)}\right] = \mathcal{F}^{(e)}\,\mathrm{e}^{i\Delta\phi}\,\mathcal{T}^{(e)}$$

Going beyond the Mauguin regime

In this talk: $\mathcal{F}^{(e,o)}$ conserved along extraordinary/ordinary rays.

Outside the Mauguin regime:

Twist of the (o) polarisation

$$\frac{\mathrm{d}}{\mathrm{d}\bar{s}}\left[\mathcal{F}^{(e)}\right] = -\,\mathcal{F}^{(o)}\,\mathrm{e}^{-i\Delta\phi}\,\mathcal{T}^{(o)}$$

Twist of the (e) polarisation

$$\frac{\mathrm{d}}{\mathrm{d}\bar{s}}\left[\mathcal{F}^{(o)}\right] = \mathcal{F}^{(e)}\,\mathrm{e}^{i\Delta\phi}\,\mathcal{T}^{(e)}$$

- If negligible twist: no cross-coupling between the two modes.
- If no deflection effects: perfect equivalence with Ong formalism

Thank you for your attention!