Simulation of polarized optical micrographs  
including light-deviation effects  
in slowly-varying birefringent structures

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Outline

1. Ray-tracing method in birefringent media
2. Validation on a simple test-case
3. Application to the visualisation of liquid crystal droplets
4. Conclusion
Motivations

Transmission of an arbitrary birefringent sample between polariser and analyzer: Jones method
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- First limitation: numerical aperture
  ⇒ generalized Jones method by Mur et al
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Transmission of an arbitrary birefringent sample between polariser and analyzer: Jones method

- First limitation: numerical aperture
  ⇒ generalized Jones method by Mur et al
- Second limitation: deflection of the extraordinary rays.

How to explain the non-zero contrast of natural light micrographs?
Objective

Question
Can we design an efficient method to simulate optical micrographs of LC samples, including light deviation effects?
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Working hypotheses: $|\nabla n| \sim \frac{1}{L} \ll \frac{1}{\lambda} + \text{Mauguin regime}$
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Goals of our method:
- Accurate simulation of Maxwell equations: WKB expansion.
Ray-tracing method in birefringent media

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- Reconstruction of bulk data
- Reconstruction of screen data, focalisation can be adjusted (as in a real microscope)
Hamiltonian ray-tracing


\[
\frac{d\eta}{ds} = \{\eta, \mathcal{H}\}
\]

\[
\eta \equiv (x, p)
\]
Hamiltonian ray-tracing


\[
\frac{d\eta}{d\bar{s}} = \{\eta, \mathcal{H}\} \\
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\]

- Discontinuity of the optical index: Fresnel boundary conditions.
Reconstruction of the electric field amplitude

\[ qV_0 < V_0 \quad qV_0 > V_0 \]
\[ \bar{s}_i = 0 \quad \bar{s}_f \]

New result:
\[ \mathcal{F}^{(\alpha)} = n_{\text{eff}} \sqrt{q} E \text{ conserved along a ray of the family } \alpha = e, o, i. \]

\[ \Rightarrow \text{Compact statement of the conservation of energy} \]
Mapping $\pi : x_i \rightarrow x_f$:

- No caustics: one-to-one correspondance
- Caustic domains: many-to-one correspondance
Caustics

Mapping $\pi : \mathbf{x}_i \rightarrow \mathbf{x}_f$:
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- Caustic domains: many-to-one correspondance

$\Rightarrow$ Necessity of finding all source points $\{\mathbf{x}_i^{(1)}, \mathbf{x}_i^{(2)}, \ldots\}$ for a given target point $\mathbf{x}_f$.

(Homotopy continuation algorithm on $\pi(\mathbf{x}_i) - \mathbf{x}_f$)
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Validation on a simple test-case

Setup

Incident plane wave on a transverse cholesteric helix: Poynting vector field $\mathbf{S}$ inside the cholesteric phase?

Initial polarisation: $\frac{e_x + e_y}{\sqrt{2}}$

Two methods of resolution:
- Our improved ray-tracing method
- Exact resolution of Maxwell Eqs. (FDTD)
Validation on a simple test-case

Results

FDTD simulation

Ray tracing simulation

Horizontal profile

Vertical profile

Fast and accurate reconstruction of far from the caustic boundaries

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Results

Fast and accurate reconstruction of $S$ far from the caustic boundaries
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Nematic twisted bipolar droplet (SSY in water)

Twisted bipolar droplet in SSY suspensions:
- Planar anchoring: 2 defects
- Double twist (giant elastic anisotropy $K_2 \ll K_{1,3}$)

Orientation in the microscope:
Computation of the natural light micrographs

Natural light micrographs: average over all polarisation states.
Results

\[ \Delta z = 0 \, \mu m \]

\[ \Delta z = 50 \, \mu m \]

Simulation

Deflection map
(extraordinary rays)
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Conclusion and outlook

- New method with fast and accurate reconstruction of $S$ far from caustics.
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- Good agreement with experimental micrographs of twisted bipolar droplets.
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- New method with fast and accurate reconstruction of $S$ far from caustics.
- Good agreement with experimental micrographs of twisted bipolar droplets.
- Perspectives:
  - CIE 1931 color space
  - Role of numerical aperture?
  - Link between chirality and symmetry-breaking in micrographs?
  - New systems: skyrmions, cholesteric fingers, banded droplets...
Going beyond the Mauguin regime

In this talk: $\mathcal{F}^{(e,o)}$ conserved along extraordinary/ordinary rays.
Going beyond the Mauguin regime

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**Outside the Mauguin regime:**

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\frac{d}{ds} F^{(e)} = - F^{(o)} e^{-i\Delta \phi} T^{(o)}
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If negligible twist: no cross-coupling between the two modes.

If no deflection effects: perfect equivalence with Ong formalism.
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Thank you for your attention!