On the pertinence of the thermomechanical model in the Lehmann rotation of cholesteric and nematic droplets

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Outline

Introduction

- State of the art
- Questions explored during my PhD

2 Thermomechanical effects of Leslie, Akopyan and Zel'dovich

- 3 Lehmann rotation of cholesteric and nematic droplets
- Importance of the thermomechanical effects in the Lehmann effect

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First observations by Lehmann



Lehmann, 1900:



- coexistence of cholesteric droplets with the isotropic fluid
- rotation of the droplets internal texture when heated from below

Leslie interpretation of the Lehmann experiment

First explanation by Leslie in 1968:



• Existence, in a cholesteric phase, of a torque on the director: $\Gamma_{\text{TM}} = \nu \ \boldsymbol{n} \times [\boldsymbol{n} \times \boldsymbol{G}]$, with ν the Leslie thermomechanical coefficient.

Leslie interpretation of the Lehmann experiment

First explanation by Leslie in 1968:



- Existence, in a cholesteric phase, of a torque on the director: $\Gamma_{\text{TM}} = \nu \ \boldsymbol{n} \times [\boldsymbol{n} \times \boldsymbol{G}]$, with ν the Leslie thermomechanical coefficient.
- As in a wind turbine, essential role of the chirality: no rotation predicted in a nematic phase.

Leslie interpretation of the Lehmann experiment

First explanation by Leslie in 1968:



Leslie paradigm

The rotation of the texture in the Lehmann experiment is due to the Leslie thermomechanical torque $\Gamma_{\rm TM}$

Oswald & Dequidt, 2008-2014:



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• Measurement of ω_m gives a value for the thermomechanical coefficient of Leslie ν .

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- The value of ν is 10 1000 too small to explain the order of magnitude of ω_d .

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- Measurement of ω_m gives a value for the thermomechanical coefficient of Leslie ν .
- The value of ν is 10 1000 too small to explain the order of magnitude of ω_d .
- ω_d and ω_m sometimes of opposite signs!



Leslie effect \neq Lehmann effect: the Leslie paradigm must be abandoned.

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Akopyan and Zel'dovich couplings

Akopyan & Zel'dovich, 1984:

- Generalization of Γ_{TM} with terms of the type $\xi(\nabla n) G$.
- Terms in ξ are allowed both in nematics and cholesterics.



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To be explored

- Clarification on the existence of these terms.
- Can we explain the Lehmann effect with these effects?
- Can we observe the Lehmann effect in twisted nematic droplets?

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• Write down the entropy production:

$$T\overset{\circ}{\sigma} = \boldsymbol{j}^{\alpha} \cdot \boldsymbol{f}^{\alpha} + \boldsymbol{j}^{\beta} \cdot \boldsymbol{f}^{\beta}.$$

 α and β relate to the behaviour under $t \rightarrow -t$

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• Generic form of the phenomenological relations:

$$j^{lpha} = L^{lpha lpha} \, f^{lpha} + L^{lpha eta} \, f^{eta}, \qquad j^{eta} = L^{eta lpha} \, f^{lpha} + L^{eta eta} \, f^{eta}$$

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• Onsager reciprocity relations:

$$\boldsymbol{L}^{\alpha\alpha} = [\boldsymbol{L}^{\alpha\alpha}]^{\mathsf{T}}, \qquad \boldsymbol{L}^{\beta\beta} = \begin{bmatrix} \boldsymbol{L}^{\beta\beta} \end{bmatrix}^{\mathsf{T}}, \qquad \boldsymbol{L}^{\alpha\beta} = -\begin{bmatrix} \boldsymbol{L}^{\beta\alpha} \end{bmatrix}^{\mathsf{T}}$$

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• Curie principle: compatibility with the symmetries of the phase

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Thermomechanical equations

Irreversible production of entropy: $T \overset{\circ}{\sigma} = -\Gamma^{(\text{neq})} \cdot \boldsymbol{\omega} - \boldsymbol{j}^{(\sigma)} \cdot \boldsymbol{G}$

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Irreversible production of entropy: $T \overset{\circ}{\sigma} = -\Gamma^{(\text{neq})} \cdot \boldsymbol{\omega} - \boldsymbol{j}^{(\sigma)} \cdot \boldsymbol{G}$ Derivation of the phenomenological equations:



This system respects the Onsager reciprocity relations.

Simplified version of the phenomenological equations

$$\Gamma_i^{\rm TM} = \xi_{ij}(\nabla n) \, G_j$$

• Tensorial expression of $\xi_{ij}(\nabla n)$ quite complicated:

$$\begin{aligned} \xi_{ij}(\nabla n) &= -\left[\nu + \bar{\xi}_2 \left(\epsilon_{kpq} \, n_k \, n_{q,p}\right)\right] \delta_{ij}^{\perp} + \bar{\xi}_1 \, n_{l,l} \, n_k \, \epsilon_{ikj} \\ &+ \bar{\xi}_3 \left(\epsilon_{ikp} \, n_k \, n_q \, n_{p,q}\right) n_j + \bar{\xi}_4 \, \epsilon_{ikp} \, n_k \left(n_{j,p} - n_{p,j}\right). \end{aligned}$$

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• We assume a simplified form for the torque Γ^{TM} $(\bar{\xi}_i = \xi)$:

$$\boldsymbol{\Gamma}^{\mathrm{TM}} = \nu \, \boldsymbol{G}^{\perp} + \xi \left(\boldsymbol{G} \cdot \boldsymbol{\nabla} \right) \boldsymbol{n}$$

- $\star~\nu :$ Leslie effect, allowed only in cholesterics.
- $\star~\xi\colon$ Akopyan & Zel'dovich effect, allowed both in nematics and cholesterics.

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Translationally invariant configurations (TIC)



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 $\omega_p \neq \omega_m$ in general \Rightarrow we can deduce $(\bar{\nu}, \xi)$ from (ω_p, ω_m) .

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Experimental setup



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Rotation of the planar TIC



Rotation of the mixed TIC



Angular velocities with a mixture of $\overline{\text{CCN}}$ -37 + 3 % CC



 $\omega_p \neq \omega_m \Rightarrow$ we can measure $\bar{\nu}$ and ξ .

Final results

From our theoretical model, we calculate just below T_{ChI} :

	CC	R811
$ar{ u}/q~({ m fN/K})$	11 ± 1	3 ± 1
$\xi~({ m fN/K})$	-35 ± 17	-25 ± 17

 $\Rightarrow (\bar{\nu}/q)_{\rm CC} \neq (\bar{\nu}/q)_{\rm R811}$ and $\xi_{\rm CC} \approx \xi_{\rm R811}$.

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 $\Rightarrow (\bar{\nu}/q)_{\rm CC} \neq (\bar{\nu}/q)_{\rm R811}$ and $\xi_{\rm CC} \approx \xi_{\rm R811}$.

- We have confirmed theoretically and experimentally the existence of the Akopyan & Zel'dovich coupling.
- Typical order of magnitude of 10 fN/K.
- $\bar{\nu}/q$ depends on the chiral dopant, contrary to ξ .

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A quick reminder



Questions

- What is the texture inside the droplets?
- Is there a scaling law for the angular velocity ω_d ?

Optical micrographs



 $\rm CCN\text{-}37$ + R811 or CC: planar anchoring at the droplet interface

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Numerical minimization of the free energy

• Unit director field:
$$n_s = \underset{n, |n|=1}{\operatorname{argmin}} F[n]$$

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Numerical minimization of the free energy

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- Discretization with Q_1 finite elements: $F[\mathbf{n}] \rightarrow f(\mathbf{N})$ with $\mathbf{N} = \begin{pmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_M \end{pmatrix}$
- Iterative minimization:
 - * $N_{(k)}$ verifying $n_{\beta} \cdot n_{\beta} = 1$
 - $\star \ \mathbf{N}_{(k+1)} = \mathcal{P}\left(\mathbf{N}_{(k)} + \boldsymbol{\delta N}\right), \text{ where } \mathcal{P} \text{ is the normalization operation } \mathbf{n}_{\beta} \to \mathbf{n}_{\beta}/\left|\mathbf{n}_{\beta}\right|$
 - $\star \delta N$ found with the truncated conjugate gradient algorithm (trust region strategy)



Essential properties of this algorithm

• $f(\mathbf{N}_{(k+1)}) < f(\mathbf{N}_{(k)})$: the energy always decreases

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- $f(\mathbf{N}_{(k+1)}) < f(\mathbf{N}_{(k)})$: the energy always decreases
- Quadratic convergence near the minimum
- Unit director field at each step: only 2M degree of freedoms in 3D

Results for a typical droplet of CCN-37



 $R = 19 \,\mu\text{m}, P = 30 \,\mu\text{m}, l_a = 0.82 \,\mu\text{m}$ (planar anchoring)

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Angular velocities



Angular velocities



Angular velocities



Data obtained with a different chiral dopant rescale on the same master curve.

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Lehmann effect

Ljubljana 19 / 32

New problematic

Rotation because of the microscopic or macroscopic chirality?

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macroscopic chirality ⇔ twisted texture (helix in at least one direction)



Lehmann effect in cholesteric droplets

New problematic

Possible tests:



chiral molecules \leftrightarrow cholesteric no macroscopic twist (compensated)

Thermal gradient \Rightarrow no rotation



no chiral molecules \leftrightarrow nematic macroscopic twist

Thermal gradient \Rightarrow rotation?

Lehmann effect in cholesteric droplets

New problematic

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chiral molecules \leftrightarrow cholesteric no macroscopic twist (compensated)

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no chiral molecules \leftrightarrow nematic macroscopic twist

Thermal gradient \Rightarrow rotation?

Question

Can we observe the Lehmann effect in droplets of a **nematic achiral phase** with a **chiral director field**?

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Lehmann effect

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Lehmann effect in nematic droplets

Stability of bipolar configuration





• Lyotropic chromonic nematic used: water + 30% SSY $(K_2/K_1 \simeq 0.16, K_2/K_3 \simeq 0.12)$



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- The sign of twist fixes the sign of the angular velocity \Rightarrow two senses of rotation



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- Achiral phase, with random handedness of the twist inside the droplets
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Rotation only due to the twist of the director field

Rotation periods



• Angular velocity $\omega_d = 2\pi/\Theta_d$ proportional to G.

Rotation periods



- Angular velocity $\omega_d = 2\pi/\Theta_d$ proportional to G.
- Period Θ_d proportional to the radius R.

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Thermomechanical model of the Lehmann effect

• Rotation "in block" of the texture without flow:

$$\frac{\mathrm{D}}{\mathrm{D}\Theta} \Big(F[\boldsymbol{n}] \Big) = 0$$

 $D/D\Theta$: change rate associated with a rotation of the texture.

Thermomechanical model of the Lehmann effect

• Rotation "in block" of the texture without flow:

$$\frac{\mathrm{D}}{\mathrm{D}\Theta} \Big(F[\boldsymbol{n}] \Big) = 0$$

 $\mathrm{D}/\mathrm{D}\Theta\text{:}$ change rate associated with a rotation of the texture.

• Using the torque equation $\Gamma^{(E)} + \Gamma^{(V)} + \Gamma^{(TM)} = 0$, we can find a prediction for the angular velocity of the droplet.

Results with nematic droplets of water + 30 % SSY



• Theoretical prediction:

$$\Theta_d \, G = \frac{R}{\xi \, L_{\xi}[\boldsymbol{n}]}$$

Results with nematic droplets of water + 30 % SSY



• Theoretical prediction:

$$\Theta_d \, G = \frac{R}{\xi \, L_{\xi}[\boldsymbol{n}]}$$

- Numerical fit: $(\xi)_{\rm fit} \approx 70 \, {\rm pN/K}$
- Measured value in CCN-37 below $T_{\rm ChI}$: $\xi \approx 30 \, {\rm fN/K}$

Qualitative agreement. Quantitative agreement?

Themomechanical effects vs. Lehmann effect

Results with cholesteric droplets of CCN-37



• Theoretical prediction:

$$\frac{\omega_d}{q\,G} = \left[\bar{\nu}/q\right] L_{\nu}[\boldsymbol{n}] + \xi \, L_{\xi}[\boldsymbol{n}]$$

Results with cholesteric droplets of CCN-37



• Theoretical prediction:

 $\frac{\omega_d}{q \, G} = \left[\bar{\nu}/q\right] L_{\nu}[\boldsymbol{n}] + \xi \, L_{\xi}[\boldsymbol{n}]$

• Numerical fit: $(\bar{\nu}/q)_{\rm fit} \approx 1.7 \,\mathrm{pN/K}$ $(\xi)_{\rm fit} \approx -3 \,\mathrm{pN/K}$ \Rightarrow values 100 bigger than those measured below $T_{\rm ChI}$ $(\sim 10 \,\mathrm{fN/K}).$

Qualitative and quantitative disagreement.

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• Experimental and theoretical confirmation of the existence of the (corrected) Akopyan & Zel'dovich coupling terms.

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- The Lehmann effect can be observed in twisted nematic droplets.
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- The Lehmann effect can be observed in twisted nematic droplets.
- The Leslie, Akopyan & Zel'dovich thermomechanical terms cannot explain the Lehmann effect.

Postdoc project

Simulation of natural and polarized light micrographs of nematic and cholesteric droplets.



- Hamiltonian ray-tracing method: $\dot{\boldsymbol{\eta}} = \boldsymbol{\Omega} \cdot \boldsymbol{\nabla}_{\boldsymbol{\eta}} (H^{e,o}),$ with $\boldsymbol{\eta} = (\boldsymbol{r}, \boldsymbol{k}).$
- Conserved quantities along a ray: $(\sqrt{q} n_{\text{eff}} E)$ and $(\sqrt{q} B)$, with q the geometrical spreading.

$$\begin{array}{ccc} \bigwedge & \uparrow & \uparrow & & \uparrow & \uparrow & \\ q > 1 & & q = 1 & & q < 1 \end{array}$$

Deviation of extraordinary rays in a cholesteric slab:



Thank you!

Conclusion

In the future...

- Why the droplets stay spherical in a temperature gradient?
- Importance of the theoretical model of A. Dequidt in the Lehmann effect?



• Convective rolls? Marangoni and/or thermohydrodynamic couplings?



Conclusion

Photobleaching experiment





- LC mixture doped with fluorescent molecules
- Gaussian beam of a laser focalized near a rotating droplet