

Improved ray-tracing for slowly varying director field:  
Simulation of optical micrographs  
of nematic and cholesteric droplets

Guilhem POY

Faculty of Physics and Mathematics, Ljubljana

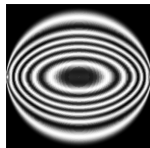
July 27, 2018

# Outline

- 1 Ray-tracing method in birefringent media
- 2 Validation on a simple test-case
- 3 Application to the visualisation of cholesteric and nematic droplets
- 4 Conclusion

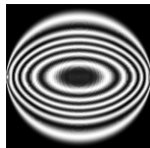
## Motivations

Transmission of an arbitrary birefringent sample  
between polariser and analyzer: Jones method



## Motivations

Transmission of an arbitrary birefringent sample between polariser and analyzer: Jones method

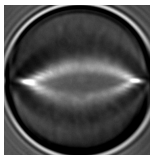


- First limitation: numerical aperture  
⇒ generalized Jones method by Mur et al

## Motivations

Transmission of an arbitrary birefringent sample between polariser and analyzer: Jones method

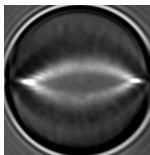
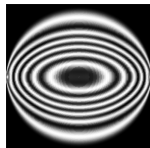
- First limitation: numerical aperture  
⇒ generalized Jones method by Mur et al
- Second limitation: deflection of the extraordinary rays.  
How to explain the non-zero contrast of natural light micrographs?



## Motivations

Transmission of an arbitrary birefringent sample between polariser and analyzer: Jones method

- First limitation: numerical aperture  
 $\Rightarrow$  generalized Jones method by Mur et al
- Second limitation: deflection of the extraordinary rays.  
 How to explain the non-zero contrast of natural light micrographs?

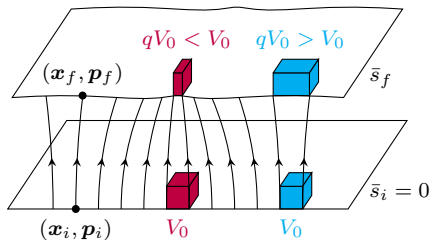


### Question

Can we design an efficient method to simulate natural light micrographs of LC samples, including light deviation effects?

## The improved ray-tracing method

Working hypotheses:  $|\nabla n| \sim \frac{1}{L} \ll \frac{1}{\lambda}$  + Mauguin regime



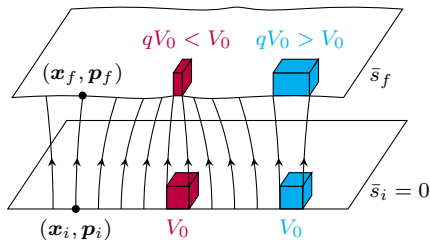
$$\frac{d\eta}{ds} = \{\eta, \mathcal{H}\}$$

$$\eta \equiv (\mathbf{x}, \mathbf{p})$$

- Evolution of extraordinary and ordinary rays: Hamilton Eqs.

## The improved ray-tracing method

Working hypotheses:  $|\nabla n| \sim \frac{1}{L} \ll \frac{1}{\lambda}$  + Mauguin regime



$$\frac{d\eta}{ds} = \{\eta, \mathcal{H}\}$$

$$\eta \equiv (\mathbf{x}, \mathbf{p})$$

- Evolution of extraordinary and ordinary rays: Hamilton Eqs.
- New result:  $n_{\text{eff}} \sqrt{q} E$  and  $\sqrt{q} B$  are conserved along a ray

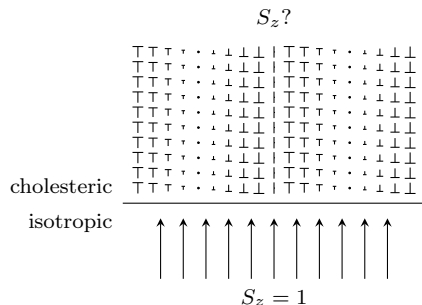


# Outline

- 1 Ray-tracing method in birefringent media
- 2 Validation on a simple test-case
- 3 Application to the visualisation of cholesteric and nematic droplets
- 4 Conclusion

## Setup

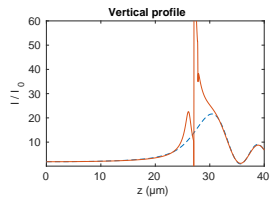
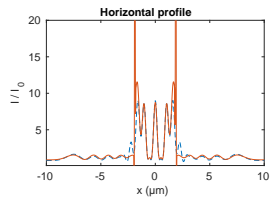
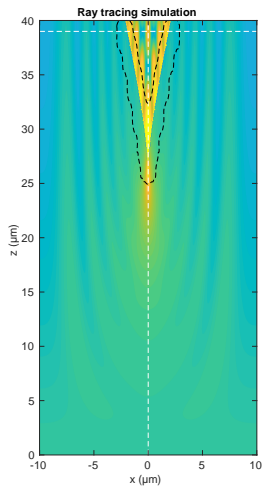
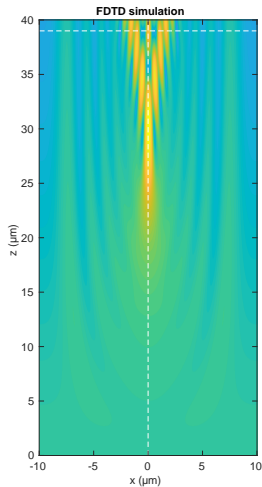
Incident plane wave on a transverse cholesteric helix:  
Poynting vector field  $\mathbf{S}$  inside the cholesteric phase?



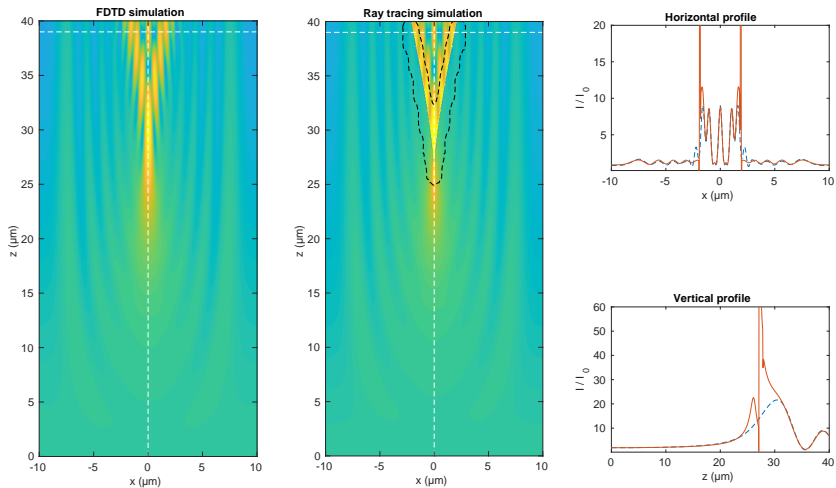
Two methods of resolution:

- Our improved ray-tracing method
- Exact resolution of Maxwell Eqs. (FDTD)

## Results



## Results

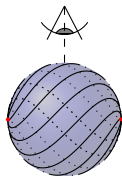
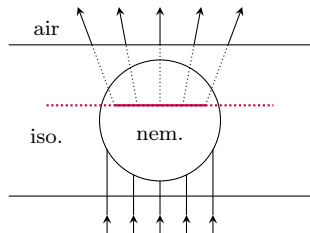


Fast and accurate reconstruction of  $\mathcal{S}$  far from the caustic boundaries

# Outline

- 1 Ray-tracing method in birefringent media
- 2 Validation on a simple test-case
- 3 Application to the visualisation of cholesteric and nematic droplets**
- 4 Conclusion

## Setup



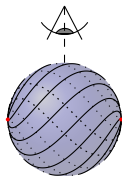
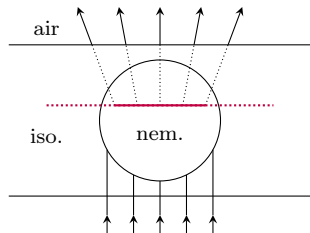
Twisted bipolar droplet:

- Planar anchoring: 2 defects
- Double twist

Two studied mixtures, with two different origin for the twist:

- CCN-37 + R811: spontaneous twist  $q_0$  of the cholesteric phase
- SSY + water: giant elastic anisotropy  $K_2 \ll K_{1,3}$

## Setup



Twisted bipolar droplet:

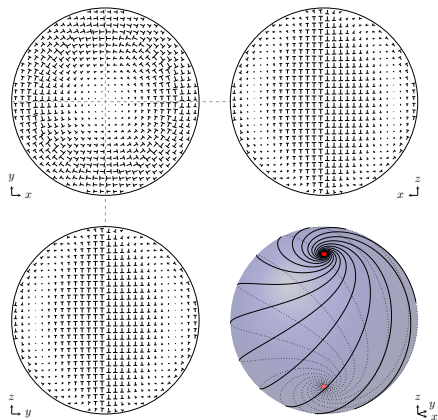
- Planar anchoring: 2 defects
- Double twist

Two studied mixtures, with two different origin for the twist:

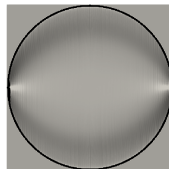
- CCN-37 + R811: spontaneous twist  $q_0$  of the cholesteric phase
- SSY + water: giant elastic anisotropy  $K_2 \ll K_{1,3}$

Natural light micrographs: average over all polarisation states.

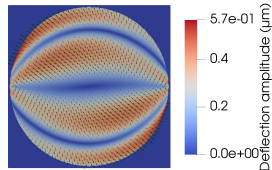
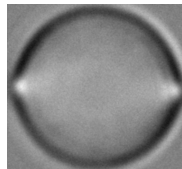
## Cholesteric twisted bipolar droplet (CCN-37+R811)



simulation



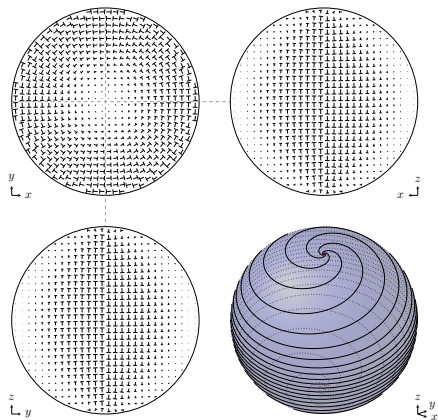
experiment



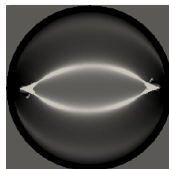
Deflection map (extraordinary rays)



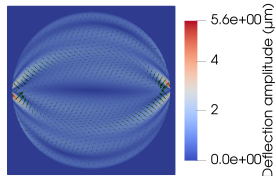
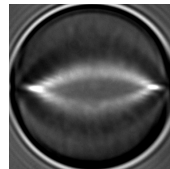
## Nematic twisted bipolar droplet (SSY in water)



simulation



experiment



Deflection map (extraordinary rays)

# Outline

- 1 Ray-tracing method in birefringent media
- 2 Validation on a simple test-case
- 3 Application to the visualisation of cholesteric and nematic droplets
- 4 Conclusion

## Conclusion and outlook

- New method with fast and accurate reconstruction of  $\mathcal{S}$  far from caustics.

## Conclusion and outlook

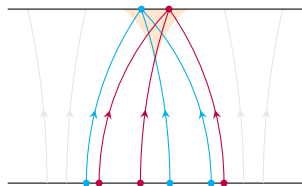
- New method with fast and accurate reconstruction of  $\mathcal{S}$  far from caustics.
- Good agreement with experimental micrographs of twisted bipolar droplets.

## Conclusion and outlook

- New method with fast and accurate reconstruction of  $\mathcal{S}$  far from caustics.
- Good agreement with experimental micrographs of twisted bipolar droplets.
- Perspectives:
  - ★ Beyond the Mauguin regime: elliptic polarisations
  - ★ Role of numerical aperture?
  - ★ Link between chirality and symmetry-breaking in micrographs?
  - ★ New systems: skyrmions, cholesteric fingers, banded droplets...

Thank you for your attention!

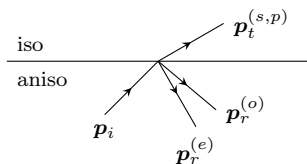
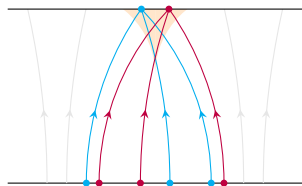
## Two sources of discontinuity



Mapping  $\mathbf{x}_i \rightarrow \mathbf{x}_f$ :

- No caustics: one-to-one correspondance
- Caustic domains: many-to-one correspondance

## Two sources of discontinuity



Mapping  $\mathbf{x}_i \rightarrow \mathbf{x}_f$ :

- No caustics: one-to-one correspondance
- Caustic domains: many-to-one correspondance

Optical index discontinuity:  
generic Fresnel boundary conditions